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Brief paper

Second-order sliding mode controller design with output constraint ☆

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ABSTRACT

The output constraints widely exist in the physical systems and severely affect the performance of the closed-loop system. This paper considers the second-order sliding mode controller design subject to an output constraint. By constructing a new barrier Lyapunov function and applying the technique of adding a power integrator, a novel second-order sliding mode control algorithm, which can be used to deal with the output constraint problem, has been developed. The proposed sliding mode algorithm enables the output variable not to violate the boundary of the constraint region. Meanwhile, it has been shown that under the output constraint the sliding variable can still be stabilized to zero in a finite time. Finally, an academic example is given to verify the feasibility of the proposed second-order sliding mode algorithm.

1. Introduction

The second-order sliding mode (SOSM) has been paid much attention in recent years, since it exhibits several merits over the conventional first-order sliding mode control (SMC). First, the SOSM provides the property of finite-time convergence for the closed-loop system (Shen, Li, Yan, Karimi, & Lam, 2018; Sun, Yun, & Li, 2017). Second, the SOSM could attenuate the chattering problem existing in the first-order SMC. This can be accomplished by taking the derivative of the first-order sliding mode controller as the virtual controller. Since the real control is the integration of the discontinuous virtual controller, the chattering can then be attenuated (Meng, Qian, & Liu, 2018; Shtessel, Edwards, Fridman, & Levant, 2013). Third, the SOSM extends the relative degree of the sliding variable in the conventional SMC from one to two. Finally, by comparing with the conventional SMC, the SOSM can also provide more accuracy for the closed-loop systems (Levant, 2007).

The concept of SOSM was first introduced in the 1980s by Emelyanov, Korovin, and Levantovsky (1986), where the twisting algorithm, as the first SOSM algorithm, is historically developed. Later, based on the concept of SOSM given in Emelyanov et al. (1986), many well-known SOSM algorithms have been proposed. The prescribed convergence law algorithm, which can be considered as the homogeneous analogue of the conventional terminal sliding mode algorithm (Feng, Yu, & Man, 2002; Man & Yu, 1997), has been developed in Levant (2007). However, it can be seen from the twisting algorithm and the prescribed convergence law algorithm that both the information of the sliding variable s and its derivative $\dot{s}$ are required. Now a question arises, are there some SOSM algorithms that do not require the use of the derivative $\dot{s}$? To answer this question, the suboptimal algorithm has been proposed in Bartolini, Ferrara, and Usai (1997), where the idea of a suboptimal algorithm is directly inspired by the time-optimal controller design for a double integrator. Meanwhile, on the basis of the twisting algorithm, the super-twisting algorithm, which solely requires the information of the sliding variable s, was developed in Levant (1993) by adding an integration to the control input. As is described by Perez Ventura and Fridman (2019), the super-twisting algorithm is effective in reducing the chattering in the presence of fast actuators while at the same time preserving the properties of the first-order sliding mode. Additionally, there still exist some other SOSM algorithms, such as quasi-continuous algorithm (Levant, 2005), Lyapunov-based algorithm (Moreno & Osorio, 2012), etc.

On the other hand, it should be pointed out that the system output constraint is a common phenomenon in practical systems due to the inherent physical limitations (see, e.g., Cheng, Park, Karimi, & Shen, 2018; Fang, Ma, Ding, & Zhao, 2019). Note that the violation of the output constraint may result in several adverse effects for control systems, such as performance degradation, serious hazards, or system damage, even system instability. Taking the electric vehicle, for example, the sideslip angle of the electric

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vehicle, as one of the most important output variables, should be constrained in a small region to keep safety (Ding, Liu, & Zheng, 2017). Another example is that the bending and torsion deformations, which are the outputs of the flapping-wing robotic aircraft, are required to incorporate the output constraints so as to prevent the structural damage (He, Mu, Chen, He, & Yu, 2018). It can be seen from the literature that the controller design problem of nonlinear systems with output constraints has gained much attention in recent years.

The barrier Lyapunov function method is an effective tool to handle the output constraint problem. A well-known method on how to construct the barrier Lyapunov function is given in Tee, Ge, and Tay (2009). It can be clearly seen that the barrier limit in Tee et al. (2009) is required to be a constant. This restriction has been relaxed in Tee, Ren, and Ge (2011), where a novel barrier function is proposed and its barrier limit varies along with the desired trajectory. Nevertheless, we can observe from the existing literature about the output constraint problems that few results are related to the sliding mode control. It should be mentioned that the barrier Lyapunov function method has recently been applied to the design of sliding mode controller in Obeid, Fridman, Laghrouche, and Harmouche (2018), where a new barrier Lyapunov function based adaptive strategy is proposed for first-order sliding mode controller design such that the proposed SMC scheme can ensure the convergence of the output variable and avoid the overestimation of the control gains. Yet the issue of output constraint problem has not been considered for the SOSM. This is because most SOSM algorithms are based on some homogeneous properties of the closed-loop system while the output constraint will violate these properties. As a matter of fact, the output constraint also significantly increases the difficulty of stability analysis. To the best of our knowledge, the problem of SOSM controller design subject to output constraint is still open.

In this paper, the SOSM control problem subject to an output constraint has been considered. A novel SOSM design scheme has been proposed in a step by step way through constructing a novel barrier Lyapunov function. First of all, it is proved by modifying the technique of adding a power integrator (Ding, Zheng, Sun, & Wang, 2018; Qian & Lin, 2001) that the sliding variable can be stabilized to the origin in a finite time. And then, it will be further verified that the output constraint condition will not be violated, which implies that the sliding variable will not escape from the constraint region. Finally, an academic example is given to show the effectiveness of the proposed SOSM method. The main contribution of the paper is two-fold. On the one hand, a novel SOSM control design method has been developed by taking the output constraint into account. It is worth pointing out that this may be the first result available currently on the problem of higher-order SMC design subject to an output constraint, although there are few results about higher-order sliding mode with state constraints given in Ding, Mei, and Li (2019), Incremona, Rubagotti, and Ferrara (2017) and Matteo and Ferrara (2010) (see Remark 3.5 for details). On the other hand, the output constraint can be preserved by constructing a novel barrier Lyapunov function, which can be accomplished by modifying the technique of adding a power integrator. As a by-product, the developed barrier Lyapunov function can also be used to design a continuous finite-time controller for a second-order system subject to output constraint.

The remainder of this paper is organized as follows. Section 2 gives the problem formulation and preliminaries. The detailed steps on how to design SOSM controller are illustrated in Section 3. Section 4 concludes this paper.

2. Problem formulation and preliminaries

In this section, a brief review for SOSM will be first given. Then, we will present the problem formulation. Finally, some important lemmas will be listed.

Consider the following nonlinear system of the form

$$\dot{x} = f(t,x) + g(t,x)u,$$
$$s = s(t,x) \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}$ are system state and control input, respectively; $f(t,x)$ and $g(t,x)$ are smooth functions; s is the output (i.e., the sliding variable) and has a relative degree $r = 2$ with respect to control input u. Additionally, the output s satisfies a constraint condition as

$$|s| < \delta \tag{2}$$

with $\delta > 0$ being any positive real number.

Remark 2.1. It is seen from system (1) that the sliding variable s is regarded as the output variable. This is reasonable because the sliding variable is usually composed of the system output and the reference signal, which are both measurable. As a matter of fact, it is a frequently-used assumption in SMC and similar conditions can be found in many studies, such as Ding, Levant, and Li (2016), Levant (2005, 2007), Park, Shen, Chang, and Lee (2018) and Qi, Zong, and Karimi (2018). Therefore, it is feasible to impose the output constraint directly on the sliding variable s.

Taking two times derivative of sliding variable s yields

$$\ddot{s} = a(t,x) + b(t,x)u \tag{3}$$

with $a(t,x) = \ddot{s}|_{u=0}$ and $b(t,x) = \frac{\partial \ddot{s}}{\partial u}$ being not exactly known, but satisfy the following assumption:

Assumption 2.1. For sliding mode dynamics (3), there exist positive constant K_m and positive function $C(x)$ such that

$$|a(t,x)| \le C(x), \quad b(t,x) \ge K_m.$$

Remark 2.2. It can be observed from the literature that almost all the SOSM algorithms are based on the following condition

$$|a(t,x)| \le C, \quad K_m \le b(t,x) \le K_M \tag{4}$$

with K_m, K_M and C being positive constants. Similar to Levant (2011), the constant assumption (4) widely used in the conventional SOSM algorithms has been extended to Assumption 2.1 in this paper. It is clearly seen that the uncertainty $a(t,x)$ is bounded by a positive function rather than a positive constant. Consequently, Assumption 2.1 is more general than the conventional assumption (4). This implies that the SMC method proposed in this paper can also be used to solve the conventional SOSM control problems.

Generally speaking, the SMC is to keep a proper chosen sliding variable s at zero by means of a high-frequency switching control. According to Levant (1993), if a discontinuous controller u can be designed such that the non-empty set $s = \dot{s} = 0$ can be kept, the motion on the set $s = \dot{s} = 0$ is called the SOSM. It can be seen from the literature that the SOSM has been widely studied in recent years, resulting in several algorithms, such as the twisting algorithm, the super-twisting algorithm (Levant, 1993), the sub-optimal algorithm (Bartolini et al., 1997), etc. However, to the best of the authors' knowledge, there are no results considering the SOSM controller design under the output constraint. The reason is that most of the existing SOSM algorithms are based on the homogeneous or geometric properties of the sliding mode dynamics. The output constraint violates the homogeneity or geometric property of the closed-loop system, and thus the existing well-known SOSM algorithms are not applicable to the control design problems under output constraint.

The purpose of this paper is to design a suitable SOSM controller for system (1) under output constraint (2) such that $s = \dot{s} = 0$ can be kept in a finite time.

Finally, we list several lemmas that serve as the basis of the key tools for the proof of the main result. To simplify notation, we denote $\lfloor x \rceil^{\alpha} = \mathrm{sign}(x)|x|^{\alpha}$.

Lemma 2.1 (Ding et al., 2016). *Assuming that $a_1 > 0$ and $0 < a_2 \leq 1$, the following holds for $\forall x, y \in R$*

$$||\lfloor x \rceil^{a_1 a_2} - \lfloor y \rceil^{a_1 a_2}| \leq 2^{1-a_2} |\lfloor x \rceil^{a_1} - \lfloor y \rceil^{a_1}|^{a_2}.$$

Lemma 2.2 (Du, Qian, Li, & Chu, 2019; Qian & Lin, 2001). *Let $\gamma(x, y) > 0$ be a function of x and y. For any positive constants a and b, one gets*

$$|x|^a |y|^b \leq \frac{a}{a+b} \gamma |x|^{a+b} + \frac{b}{a+b} \gamma^{-\frac{a}{b}} |y|^{a+b}.$$

Lemma 2.3 (Hardy, Littlewood, & Polya, 1952). *For any $c \in (0, 1)$, the following inequality holds for $\forall x_i \in R, i = 1, \ldots, n$*

$$(|x_1| + \cdots |x_n|)^c \leq |x_1|^c + \cdots + |x_n|^c$$

3. Main result

In this section, by constructing a novel barrier Lyapunov function, we will propose an SOSM controller for system (1) under the output constraint condition (2) in a step by step way. The following theorem is the main result of the paper.

Theorem 3.1. *Considering system (1) with the initial value such that the sliding variable s is inside the constraint (2) at the beginning, there exist two positive constants β_1 and β_2 such that the following SOSM controller*

$$u = -\beta_2 \cdot \Phi(s) \cdot \left\lfloor \lfloor \dot{s} \rceil^{a/r_2} + \beta_1 \lfloor s \rceil^{a/r_1} \right\rceil^{r_3/a} - \frac{C(x)}{K_m} \cdot \mathrm{sign}(\lfloor \dot{s} \rceil^{a/r_2} + \beta_1 \lfloor s \rceil^{a/r_1}) \tag{5}$$

with $\Phi(s) = \sec^2\left(\frac{\pi |s|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right)$, $\rho \geq a \geq r_1, r_2 = r_1 - \tau > 0, r_3 = r_2 - \tau \geq 0, \tau > 0$, provides for the finite-time establishment of SOSM $s = \dot{s} = 0$ in system (1), while the sliding variable s satisfies the constraint (2) for all $t \geq 0$.

Proof. By letting $s_1 = s$ and $s_2 = \dot{s}$, system (3) can be rewritten as

$$\dot{s}_1 = s_2$$
$$\dot{s}_2 = a(t, x) + b(t, x)u. \tag{6}$$

To prove Theorem 3.1, we only need to prove that system (6) will be finite-time stabilized to the origin under the constraint $|s_1| < \delta$, provided that the initial state of s_1 is located within the constraint $|s_1| < \delta$.

Step 1. Select a Lyapunov function as

$$V_1(s_1) = \frac{2\delta^{\frac{2\rho+\tau}{r_1}}}{\frac{2\rho+\tau}{r_1}\pi} \cdot \tan\left(\frac{\pi |s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right) \tag{7}$$

where ρ, r_1 and τ are real numbers satisfying $\rho \geq r_1 > 0$ and $\tau > 0$. It should be noted that the Lyapunov function $V_1(s_1)$ is defined on the region $\mathcal{D}_1 = \{s_1 : |s_1| < \delta\}$. Taking derivative of $V_1(s_1)$ along system (6) gives

$$\dot{V}_1(s_1) = \lfloor s_1 \rceil^{\frac{2\rho+\tau}{r_1}-1} \cdot s_2 \cdot \sec^2\left(\frac{\pi |s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right). \tag{8}$$

Let $\Phi(s_1) = \sec^2\left(\frac{\pi |s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right)$. Eq. (8) implies

$$\begin{aligned}
&\dot{V}_1(s_1)\\
&= \lfloor s_1 \rceil^{\frac{2\rho-r_2}{r_1}} \cdot s_2 \cdot \Phi(s_1)\\
&= \lfloor s_1 \rceil^{\frac{2\rho-r_2}{r_1}} \Phi(s_1)(s_2 - s_2^*) + \lfloor s_1 \rceil^{\frac{2\rho-r_2}{r_1}} \Phi(s_1) s_2^*
\end{aligned} \tag{9}$$

where $r_2 = r_1 - \tau > 0$ and s_2^* is a virtual control law to be determined later.

We construct the virtual controller as $s_2^* = -\beta_1 \lfloor \xi_1 \rceil^{r_2/a}$ with $\beta_1 > 0, r_1 \leq a \leq \rho$ and $\xi_1 = \lfloor s_1 \rceil^{a/r_1}$. It can be directly deduced from (9) that

$$\dot{V}_1(s_1) \leq -\beta_1 \cdot \Phi(s_1) \cdot |\xi_1|^{\frac{2\rho}{a}} + \lfloor \xi_1 \rceil^{\frac{2\rho-r_2}{a}} \cdot \Phi(s_1) \cdot (s_2 - s_2^*). \tag{10}$$

Step 2. Consider the following function

$$\begin{aligned}
&V_2(s_1, s_2) = V_1(s_1) + W_2(s_1, s_2),\\
&W_2(s_1, s_2) = \int_{s_2^*}^{s_2} \left\lfloor \lfloor \kappa \rceil^{a/r_2} - \lfloor s_2^* \rceil^{a/r_2} \right\rceil^{\frac{2\rho-r_3}{a}} d\kappa.
\end{aligned} \tag{11}$$

with $r_3 = r_2 - \tau \geq 0$, which is defined on the region $\mathcal{D}_2 = \mathcal{D}_1 \times \mathbb{R}$. According to the Propositions B.1 and B.2 in the Ref. Qian and Lin (2001), it can be verified that the function $V_2(s_1, s_2)$ is C^1 and positive definite on the region $\mathcal{D}_2$. And thus the function $V_2(s_1, s_2)$ can be considered as a Lyapunov function on the region $\mathcal{D}_2$. Differentiating the Lyapunov function candidate $V_2(s_1, s_2)$ along system (6) gets

$$\begin{aligned}
&\dot{V}_2(s_1, s_2)\\
&= \dot{V}_1(s_1) + \frac{\partial W_2(s_1, s_2)}{\partial s_1}\dot{s}_1 + \frac{\partial W_2(s_1, s_2)}{\partial s_2}\dot{s}_2\\
&\leq -\beta_1 \cdot \Phi(s_1) \cdot |\xi_1|^{2\rho/a} + \lfloor \xi_1 \rceil^{\frac{2\rho-r_2}{a}} \cdot \Phi(s_1) \cdot (s_2 - s_2^*)\\
&\quad + \frac{\partial W_2(s_1, s_2)}{\partial s_1}\dot{s}_1 + \lfloor \xi_2 \rceil^{\frac{2\rho-r_3}{a}} \dot{s}_2
\end{aligned} \tag{12}$$

with $\xi_2 = \lfloor s_2 \rceil^{a/r_2} - \lfloor s_2^* \rceil^{a/r_2}$.

In the following, we will estimate the terms in the right hand side of (12).

With the fact $0 < r_1/a \leq 1$ and

$$|s_2 - s_2^*| = \left| \lfloor s_2 \rceil^{\frac{a}{r_2} \times \frac{r_2}{a}} - \lfloor s_2^* \rceil^{\frac{a}{r_2} \times \frac{r_2}{a}} \right|,$$

applying Lemma 2.1 gives

$$\lfloor \xi_1 \rceil^{\frac{2\rho-r_2}{a}} (s_2 - s_2^*) \leq 2^{1-r_2/a} |\xi_1|^{\frac{2\rho-r_2}{a}} |\xi_2|^{r_2/a}. \tag{13}$$

Next by using Lemma 2.2 and (13), we get

$$\begin{aligned}
&\lfloor \xi_1 \rceil^{\frac{2\rho-r_2}{a}} (s_2 - s_2^*) \cdot \Phi(s_1)\\
&\leq \frac{1}{4}\beta_1 |\xi_1|^{2\rho/a} \cdot \Phi(s_1) + c_1(\beta_1) |\xi_2|^{2\rho/a} \cdot \Phi(s_1)
\end{aligned} \tag{14}$$

where $c_1(\beta_1)$ is defined as

$$c_1(\beta_1) = 2^{-\frac{r_2}{a}} \frac{r_2}{\rho} \left(\frac{(2\rho - r_2) 2^{2-r_2/a}}{\rho \beta_1}\right)^{\frac{2\rho-r_2}{r_2}}.$$

On the other hand, one obtains from Lemma 2.1 that

$$\begin{aligned}&\left|\frac{\partial W_2(s_1,s_2)}{\partial s_1}\dot{s}_1\right|\\&\le \frac{2\rho-r_3}{a}|s_2-s_2^*\|\xi_2|^{\frac{2\rho-r_3}{a}-1}\left|\frac{\partial\lfloor s_2^*\rceil^{a/r_2}}{\partial s_1}\dot{s}_1\right|\\&\le \frac{2\rho-r_3}{a}\left|\lfloor s_2\rceil^{\frac{a}{r_2}\times\frac{r_2}{a}}-\lfloor s_2^*\rceil^{\frac{a}{r_2}\times\frac{r_2}{a}}\right|\\&\quad\times|\xi_2|^{\frac{2\rho-r_3}{a}-1}\left|\frac{\partial\lfloor s_2^*\rceil^{a/r_2}}{\partial s_1}\dot{s}_1\right|\\&\le \frac{2\rho-r_3}{a}2^{1-r_2/a}|\xi_2|^{\frac{r_2}{a}+\frac{2\rho-r_3}{a}-1}\left|\frac{\partial\lfloor s_2^*\rceil^{a/r_2}}{\partial s_1}\dot{s}_1\right|. \quad (15)\end{aligned}$$

Taking $s_2^*=-\beta_1\lfloor\xi_1\rceil^{r_2/a}$ into account, it can be shown that

$$\begin{aligned}\left|\frac{\partial(\lfloor s_2^*\rceil^{a/r_2})}{\partial s_1}\right|&=\beta_1^{a/r_2}|\frac{\partial\xi_1}{\partial s_1}|\\&=\frac{a\beta_1^{a/r_2}}{r_1}|s_1|^{a/r_1-1}. \quad (16)\end{aligned}$$

By using Lemma 2.3, it is not difficult to give

$$|s_2|=\left|\xi_2-\beta_1^{a/r_2}\xi_1\right|^{r_2/a}\le\left(|\xi_2|^{\frac{r_2}{a}}+\beta_1|\xi_1|^{\frac{r_2}{a}}\right). \quad (17)$$

With the help of Eq. (6), by combining with (16) and (17), one gets

$$\begin{aligned}&\left|\frac{\partial(\lfloor s_2^*\rceil^{a/r_2})}{\partial s_1}\dot{s}_1\right|\\&=\left|\frac{\partial(\lfloor s_2^*\rceil^{a/r_2})}{\partial s_1}s_2\right|\\&\le\frac{a\beta_1^{a/r_2}}{r_1}|\xi_1|^{1-r_1/a}\times\left(|\xi_2|^{\frac{r_2}{a}}+\beta_1|\xi_1|^{\frac{r_2}{a}}\right). \quad (18)\end{aligned}$$

By Lemma 2.2, we can obtain from (18) that there exist two positive constants $c_2(\beta_1)$ and $c_3(\beta_1)$ such that

$$\left|\frac{\partial(\lfloor s_2^*\rceil^{a/r_2})}{\partial s_1}\dot{s}_1\right|\le c_2(\beta_1)|\xi_1|^{1-\tau/a}+c_3(\beta_1)|\xi_2|^{1-\tau/a} \quad (19)$$

with $c_2(\beta_1)=\frac{a\beta_1^{1+a/r_2}}{r_1}+\frac{a\beta_1^{a/r_2}}{r_1}\frac{a-r_1}{a-\tau}$ and $c_3(\beta_1)=\frac{a\beta_1^{a/r_2}}{r_1}\frac{r_2}{a-\tau}$.

Substituting (19) into (15) obtains

$$\begin{aligned}&\left|\frac{\partial W_2(s_1,s_2)}{\partial s_1}\dot{s}_1\right|\le\frac{2^{1-r_2/a}(2\rho-r_3)}{a}|\xi_2|^{\frac{r_2}{a}+\frac{2\rho-r_3}{a}-1}\times\\&\left(c_2(\beta_1)|\xi_1|^{1-\tau/a}+c_3(\beta_1)|\xi_2|^{1-\tau/a}\right). \quad (20)\end{aligned}$$

Note that $\Phi(s_1)\ge 1$. Using Lemma 2.2 once again implies

$$\begin{aligned}\left|\frac{\partial W_2(s_1,s_2)}{\partial s_1}\dot{s}_1\right|&\le\frac{1}{4}\beta_1\cdot\Phi(s_1)\cdot|\xi_1|^{2\rho/a}\\&\quad+c_4(\beta_1)\cdot\Phi(s_1)\cdot|\xi_2|^{2\rho/a} \quad (21)\end{aligned}$$

where $c_4(\beta_1)$ is defined as

$$\begin{aligned}c_4(\beta_1)=&\frac{2^{-r_2/a}}{a\rho}(2\rho-r_3)(2\rho+\tau-a)c_2(\beta_1)\times\\&\left(\frac{2^{2-r_2/a}(2\rho-r_3)(a-\tau)c_2(\beta_1)}{\rho a\beta_1}\right)^{\frac{a-\tau}{2\rho+\tau-a}}\\&+\frac{2^{1-r_2/a}(2\rho-r_3)}{a}c_3(\beta_1).\end{aligned}$$

Putting (14) and (21) into (12) gets

$$\begin{aligned}\dot{V}_2(s_1,s_2)\le&-\tfrac{\beta_1}{2}\cdot\Phi(s_1)\cdot|\xi_1|^{2\rho/a}\\&+[c_1(\beta_1)+c_4(\beta_1)]\cdot\Phi(s_1)|\xi_2|^{2\rho/a}+\lfloor\xi_2\rceil^{\frac{2\rho-r_3}{a}}\dot{s}_2.\end{aligned}$$

This, together with Assumption 2.1 and the fact $\dot{s}_2=a(t,x)+b(t,x)u$, implies

$$\begin{aligned}\dot{V}_2(s_1,s_2)\le&-\tfrac{\beta_1}{2}\cdot\Phi(s_1)\cdot|\xi_1|^{2\rho/a}\\&+[c_1(\beta_1)+c_4(\beta_1)]\cdot\Phi(s_1)|\xi_2|^{2\rho/a}\\&+|\xi_2|^{\frac{2\rho-r_3}{a}}C(x)+\lfloor\xi_2\rceil^{\frac{2\rho-r_3}{a}}\cdot b(t,x)u. \end{aligned}\quad (22)$$

The SOSM controller can be designed as

$$u=-\beta_2\cdot\Phi(s_1)\cdot\lfloor\xi_2\rceil^{r_3/a}-\frac{C(x)}{K_m}\cdot\text{sign}(\xi_2) \quad (23)$$

with $\beta_2\ge\frac{c_1(\beta_1)+c_4(\beta_1)+\frac{\beta_1}{2}}{K_m}$. Substituting controller (23) into (22), we have for all $(s_1,s_2)\in\mathcal{D}_2$

$$\dot{V}_2(s_1,s_2)\le-\tfrac{\beta_1}{2}\cdot\Phi(s_1)\cdot(|\xi_1|^{2\rho/a}+|\xi_2|^{2\rho/a}). \quad (24)$$

Next, we will prove the finite-time stability of the closed-loop system (6) and (23).

Note that $2x\ge\tan(x)$ for any $x\in(0,\frac{\pi}{4})$. This property implies that there exists a small region of the origin

$$\Omega_1=\left\{(s_1,s_2):\frac{\pi|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}<\frac{\pi}{4},s_2\in R\right\}$$

such that for all $(s_1,s_2)\in\Omega_1\subset\mathcal{D}_2$

$$V_2(s_1,s_2)\le\frac{2r_1|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\rho+\tau}+\int_{s_2^*}^{s_2}\left\lfloor\lfloor\kappa\rceil^{\frac{a}{r_2}}-\lfloor s_2^*\rceil^{\frac{a}{r_2}}\right\rceil^{\frac{2\rho-r_3}{a}}d\kappa.$$

By a simple calculation, we have

$$\begin{aligned}&\int_{s_2^*}^{s_2}\left\lfloor\lfloor\kappa\rceil^{a/r_2}-\lfloor s_2^*\rceil^{a/r_2}\right\rceil^{\frac{2\rho-r_3}{a}}d\kappa\\&\le|s_2-s_2^*\|\xi_2|^{\frac{2\rho-r_3}{a}}\\&\le 2^{1-r_2/a}|\xi_2|^{r_2/a}|\xi_2|^{\frac{2\rho-r_3}{a}}\\&=2^{1-r_2/a}|\xi_2|^{\frac{2\rho+\tau}{a}}.\end{aligned}$$

Let $\lambda_0=\max\left\{\frac{2r_1}{2\rho+\tau},2^{1-r_2/a}\right\}$. It is clear that

$$V_2(s_1,s_2)\le\lambda_0(|\xi_1|^{\frac{2\rho+\tau}{a}}+|\xi_2|^{\frac{2\rho+\tau}{a}}).$$

Choosing $c=\frac{\beta_1}{4\times\lambda_0^{\frac{2\rho}{2\rho+\tau}}}$, it can be verified from the fact $\Phi(s_1)\ge 1$ that for $\forall(s_1,s_2)\in\Omega_1\subset\mathcal{D}_2$

$$\dot{V}_2(s_1,s_2)+cV_2^{\frac{2\rho}{2\rho+\tau}}(s_1,s_2)\le 0. \quad (25)$$

Note that $\frac{2\rho}{2\rho+\tau}\in(0,1)$. It follows from the finite-time Lyapunov theory given in Bhat and Bernstein (2000) that system (6) defined on the region $\mathcal{D}_2$ can be finite-time stabilized by controller (23).

Finally, we will verify the boundedness of the sliding variable s_1.

By taking $V_2(s_1,s_2)=V_1(s_1)+W_2(s_1,s_2)$ and (24) into account, we have for all $(s_1,s_2)\in\mathcal{D}_2$

$$V_1(s_1(t))\le V_2(s_1(t),s_2(t))\le V_2(s_1(0),s_2(0)).$$

In view of the definition of $V_1(s_1)$, one obtains

$$\frac{2\delta^{\frac{2\rho+\tau}{r_1}}}{\frac{2\rho+\tau}{r_1}\pi}\cdot\tan\left(\frac{\pi|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right)\le V_2(s_1(0),s_2(0)),$$

which can be rewritten as

$$\tan\left(\frac{\pi|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right) \le \frac{\frac{2\rho+\tau}{r_1}\pi}{2\delta^{\frac{2\rho+\tau}{r_1}}}\cdot V_2(s_1(0),s_2(0)).$$

This implies

$$\left(\frac{\pi|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right) \le \arctan\left(\frac{\frac{2\rho+\tau}{r_1}\pi}{2\delta^{\frac{2\rho+\tau}{r_1}}}\cdot V_2(s_1(0),s_2(0))\right). \tag{26}$$

Apparently, it can be obtained from (26) that

$$\frac{\pi|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}} < \frac{\pi}{2},$$

which directly leads to $|s_1| < \delta$.

Note that controller (23) can be rewritten as (5). This implies that controller (5) finite-time establishes the SOSM $s = \dot{s} = 0$ in system (1) under constraint (2). This completes the proof. ■

Remark 3.1. It can be seen from (5) that when the sliding variable s approaches δ, the gain function $\Phi(s)$ will become arbitrarily large such that the sliding mode controller will provide sufficient control effort to pull the sliding variable s back from the boundary of the constraint $|s| < \delta$. This implies that the proposed controller (5) is not a bounded control.

Remark 3.2. The reason to choose the parameters ρ, τ, a satisfying $\rho \ge a \ge r_1$ is to avoid the singularity of $\dot{V}_2(s_1, s_2)$. It can be clearly seen from (11) that if $\rho < a$ or $a < r_1$, the derivative of $V_2(s_1, s_2)$ may be singular, since the fractional power $\frac{2\rho - r_3}{a}$ may be less than one. Meanwhile, when the sliding variables are far away from the origin, it can be verified from (5) that the bigger ρ or a means more control effort, which will improve the convergence performance of the closed-loop system. On the other hand, the parameters r_1, r_2, r_3 are required to satisfy $r_2 = r_1 - \tau > 0$ and $r_3 = r_2 - \tau \ge 0$. Actually, this condition ensures the local homogeneity of $V_2(s_1, s_2)$ and $\dot{V}_2(s_1, s_2)$, which is a sufficient condition for finite-time convergence of the sliding variables.

Remark 3.3. The key technique used in the paper is the construction of a novel barrier Lyapunov function to solve the output constraint problem. As a matter of fact, if we do not consider the constraint problem, the Lyapunov function $V_1(s_1)$ can be chosen as

$$V_1(s_1) = \frac{r_1|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\rho+\tau}. \tag{27}$$

Imposing a tangent function on (27) yields the Lyapunov function (7), which enables the sliding variable s_1 not to escape from the region $\{s_1 : |s_1| < \delta\}$. This is because when the sliding variable s_1 approaches its boundary $|s_1| = \delta$, the gain function $\Phi(s_1) = \sec^2\left(\frac{\pi|s_1|^{\frac{2\rho+\tau}{r_1}}}{2\delta^{\frac{2\rho+\tau}{r_1}}}\right)$ of controller (5) increases too, which forces the sliding variable to stay in the restricted region $\{s_1 : |s_1| < \delta\}$.

Remark 3.4. From the definition of $\Phi(s)$, one has $\lim_{\delta\to\infty}\Phi(s) = 1$ for all $|s| < \delta$. And then controller (5) can be rewritten as

$$\begin{aligned} u = &-\beta_2 \cdot \left\lfloor \lfloor\dot{s}\rceil^{a/r_2} + \beta_1\lfloor s\rceil^{a/r_1}\right\rceil^{r_3/a} \\ &- \frac{C(x)}{K_m}\cdot \mathrm{sign}(\lfloor\dot{s}\rceil^{a/r_2} + \beta_1\lfloor s\rceil^{a/r_1}) \end{aligned} \tag{28}$$

This implies that, if δ is sufficiently large, the constraint (2) will be released and controller (28) can be considered as the conventional SOSM controller for system (1) without constraint (2).

Remark 3.5. It should be noted that the constrained control of nonlinear systems by using SOSM has also been considered in Ding et al. (2019), Incremona et al. (2017) and Matteo and Ferrara (2010). It can be seen from Matteo and Ferrara (2010) that the problem of the robust control of a double integrator subject to both state and control input constraints is considered. The proposed SOSM controller in Matteo and Ferrara (2010) can steer the system states to the origin in a finite time and keep the states inside the given constraint. And then, the method in Matteo and Ferrara (2010) has been extended in Ding et al. (2019) and Incremona et al. (2017), where the design of SMC algorithm for nonlinear systems in the presence of hard inequality constraints on both control and state variables is proposed and the maximization of the domain of attraction is discussed. However, it can be seen from the proposed sliding mode algorithms given in Ding et al. (2019), Incremona et al. (2017) and Matteo and Ferrara (2010) that the convergence can be assured only in a small domain of attraction of a preset region, while the sliding variables in this paper can finite-time converge to the origin from any initial points in the given region. It implies that the methods given in Ding et al. (2019), Incremona et al. (2017) and Matteo and Ferrara (2010) cannot be applied to the systems considered in this paper.

It can be seen from (5) that there are two terms (i.e., a continuous term and a discontinuous term) included in the derived SOSM controller. The discontinuous term is used to eliminate the adverse effect of uncertainties, while the continuous term is utilized to guarantee the property of finite-time convergence. As a matter of fact, when $r_3 = 0$ the following relation holds:

$$\left\lfloor \lfloor\dot{s}\rceil^{a/r_2} + \beta_1\lfloor s\rceil^{\frac{a}{r_1}}\right\rceil^0 = \mathrm{sign}\left(\lfloor\dot{s}\rceil^{\frac{a}{r_2}} + \beta_1\lfloor s\rceil^{\frac{a}{r_1}}\right).$$

Under this circumstance, with $r_3 = r_2 - \tau = 0$ and $r_2 = r_1 - \tau$ in mind, controller (5) reduces to a simper version as

$$u = -\left(\beta_2 + \frac{C(x)}{K_m}\right)\cdot\Phi(s)\cdot \mathrm{sign}(\lfloor\dot{s}\rceil^2 + \beta_1\cdot s) \tag{29}$$

Then, we have the following result.

Corollary 3.1. *There exist two positive constants β_1 and β_2 such that controller (29) provides for the finite-time establishment of SOSM $s = \dot{s} = 0$ in system (1). Meanwhile, the sliding variable s satisfies the constraint condition (2) for all $t \ge 0$, provided that the sliding variable s is located in the constraint $|s| < \delta$ at the beginning.*

Remark 3.6. It can be seen from the proof of Theorem 3.1 that the parameter β_1 can be any positive constant. However, it cannot be chosen to be very small, because it affects the convergence of the sliding variable s. The larger β_1 implies the rapid convergence performance of the sliding variable. On the other hand, we know that the parameter β_2 should satisfy $\beta_2 \ge \frac{c_1(\beta_1)+c_4(\beta_1)+\frac{\beta_1}{2}}{K_m}$. It seems that the parameter β_2 should be greater than a large constant. This is because the control design in this paper adopts a backstepping-like technique, which overestimates the value of β_2. As a matter of fact, we could choose some smaller values for the parameter β_2.

Example 3.1. Consider the following nonlinear system:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^3 \\ \dot{x}_2 &= f(t,x) + g(t,x)u \end{aligned} \tag{30}$$

with $|f(t,x)| \le 1 + x_1^2$ and $g(t,x) \ge 1$. The state constraint is given as

$$|x_1(t)| < 2. \tag{31}$$

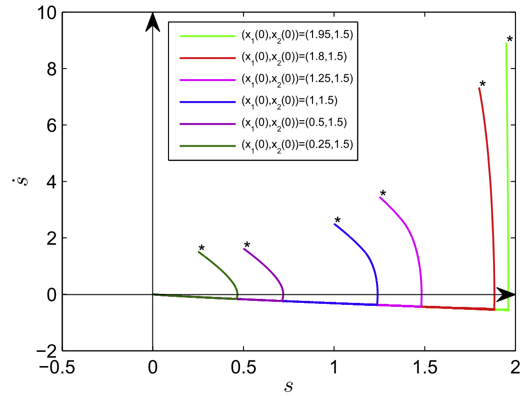

Fig. 1. The phase plot of $s - \dot{s}$ under SOSM controller (34).

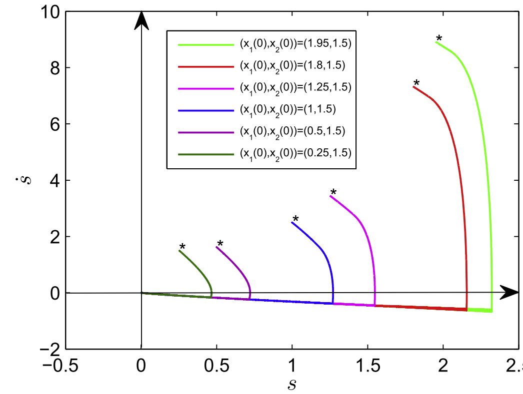

Fig. 4. The phase plot of $s - \dot{s}$ under SOSM controller (35).

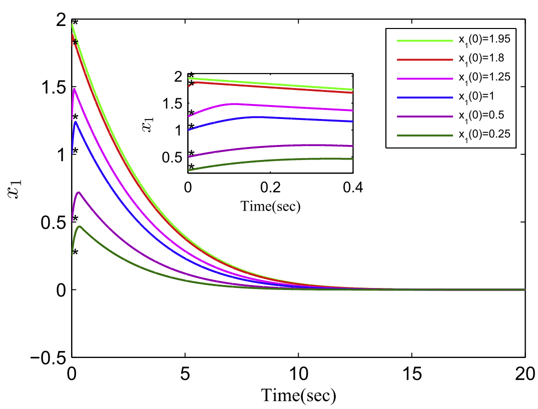

Fig. 2. Time history of x_1 under SOSM controller (34).

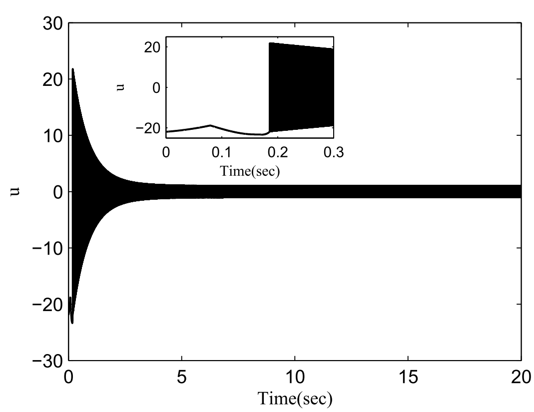

Fig. 3. Time history of controller (34) under initial condition $(x_1(0)), x_2(0) = (1, 1.5)$.

The goal is to design a controller such that the state x_1 will converge to zero under the constraint (31).

Let $s = x_1$. From (30), we have the following sliding mode dynamics as

$$\ddot{s} = a(t, x) + b(t, x)u \tag{32}$$

with $a(t, x) = f(t, x) + 3x_1^2(x_2 + x_1^3)$ and $b(t, x) = g(t, x)$. This implies that we can take

$$C(x) = 1 + x_1^2 + 3x_1^2(|x_2| + |x_1|^3), K_m = 1. \tag{33}$$

With the state restriction (31) in mind, the restriction imposed on the sliding variable s is $|s| < \delta = 2$.

According to Theorem 3.1, if we let $\rho = a = r_1 = 1$, $\tau = 1/6$, the SOSM controller can be designed as

$$u = -\beta_2 \cdot \Phi(s) \cdot \left\lfloor \lfloor \dot{s} \rceil^{6/5} + \beta_1 s \right\rceil^{2/3} - \frac{C(x)}{K_m} \cdot \text{sign}(\lfloor \dot{s} \rceil^{6/5} + \beta_1 s) \tag{34}$$

with $\Phi(s) = \sec^2\left(\frac{\pi |s|^{\frac{13}{6}}}{2^{\frac{19}{6}}}\right)$.

Meanwhile, according to Remark 3.4, we also construct a conventional SOSM controller without the output constraint to show the comparison

$$u = -\beta_2 \cdot \left\lfloor \lfloor \dot{s} \rceil^{6/5} + \beta_1 s \right\rceil^{2/3} - \frac{C(x)}{K_m} \cdot \text{sign}(\lfloor \dot{s} \rceil^{6/5} + \beta_1 s). \tag{35}$$

The gains $C(x)$ and K_m of controller (34) are chosen as (33). Other parameters are chosen as $\beta_1 = 0.25$, $\beta_2 = 5$. The parameters of controller (35) are the same to controller (34). Additionally, the uncertainties are assumed to be $f(t, x) = \sin(x_2) + x_1^2$ and $g(t, x) = 1 + 0.5(\sin(t))^2$. The initial states are chosen as $(x_1(0), x_2(0)) = (1.95, 1.5)$, $(1.8, 1.5)$, $(1.25, 1.5)$, $(1, 1.5)$, $(0.5, 1.5)$, $(0.25, 1.5)$, respectively. The sampling time is taken as 0.0001 s and the simulation results are given in Figs. 1–3.

Fig. 1 shows the phase plot of $s - \dot{s}$ under different initial states, while Fig. 2 gives the time history of x_1. It can be seen from Figs. 1–2 that the sliding variables will converge to the origin, but the state x_1 will not cross the boundary $|x_1| = \delta$. As a matter of fact, it can be seen from Fig. 2 that when the state x_1 approaches its boundary, it will be traced back. This is because when the state x_1 approaches the boundary $|x_1| = \delta$, the gain function $\Phi(s)$ increases too, which provides more powerful control effort. This property can also be verified by Fig. 3, which shows the time history of controller (34) under the initial state $(1, 1.5)$. It can be clearly observed from Fig. 3 that the control input signal is getting bigger as the state x_1 approaches the boundary, which verifies the theoretical analysis.

Meanwhile, the simulation results under SOSM controller (35) are also given to show the performance without output constraint. The simulation results are given in Figs. 4–5. It can be clearly observed that under some initial states, the state x_1 under SOSM controller (35) will cross the boundary of the restriction. These simulation results verify the merits of the proposed SOSM controller (34).

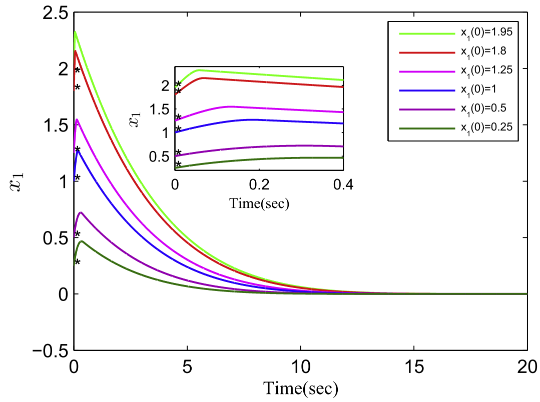

Fig. 5. Time history of x_1 under SOSM controller (35).

4. Conclusion

By constructing a novel tangent-type barrier Lyapunov function, a new SOSM control design method has been proposed in this paper to handle the output constraint problem. The control design is carried out by using the modified version of adding a power integrator technique. It has been shown that the proposed SOSM controller provides the finite-time stability of the closed-loop sliding mode control systems. The future work will be focused on extending the result to the higher-order ($n \geq 3$) sliding mode dynamics.

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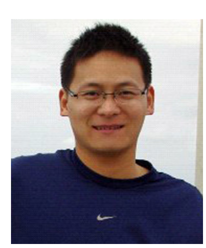

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