

On the vibration control capability of shape memory alloy composite beams



A.R. Damanpack, M. Bodaghi, M.M. Aghdam^{*}, M. Shakeri

Thermo-elasticity Center of Excellence, Department of Mechanical Engineering, Amirkabir University of Technology, Tehran, Iran

ARTICLE INFO

Article history:

Available online 21 December 2013

Keywords:

Shape memory alloys
SMA composite beams
Material and geometrical non-linearity
Active/passive vibration control

ABSTRACT

In this paper, the vibration control capability of shape memory alloy (SMA) composite beams subjected to impulsive loads is examined. In order to simulate the SMA response, a one-dimensional constitutive model is introduced which is able to reproduce pseudo-elasticity, martensite transformation/orientation and in particular ferro-elasticity effects. A numerical algorithm is presented to solve non-linear SMA constitutive model by means of an elastic-predictor inelastic-corrector return map procedure. The equivalent single layer theory of Rayleigh–Euler–Bernoulli is used to describe displacement field of SMA laminated composite beams. Geometrical non-linearity is also considered in the von Karman sense. Considering rotary inertia effects, finite element equations of motion are developed using the Hamilton principle. Newmark and Newton–Raphson methods are utilized to obtain an incremental solution of the problem. Extensive numerical results are presented to provide an insight into the influence of pre-strain, temperature, location and thickness of SMA layers on the vibration control of SMA composite beams subjected to various blast pulses. Considering ferro-elasticity effect, results reveal the fact that SMA layers with high pre-strain have a passive vibration control capability in low temperatures and yield a better efficiency in comparison with pseudo-elastic SMA layers.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

Smart hybrid composites with either embedded or surface mounted shape memory alloy (SMA) components such as fibers, wires, ribbons and thin films have proved to be unique material systems that provide tremendous potential for creating new paradigms for material–structural interaction [1,2]. SMAs are materials capable of undergoing large recoverable strains in the order of 10% while producing hysteresis. This unique behavior is a result of the so-called martensitic phase transformation between austenite (A) and martensite (M) in response of stress and/or temperature changes [2]. Furthermore, each martensitic crystal formed can have a different orientation direction, called a variant. The assembly of martensitic variants may exist in two different forms: multiple-variant martensite or twinned martensite (M^t) and single-variant martensite or detwinned martensite (M^d). At high temperatures, SMAs may recover large deformations during mechanical loading–unloading patterns (pseudo-elasticity, PE). At low temperatures, a mechanical loading–unloading results in a residual strain which can be recovered by thermal loadings (shape memory effect, SME) or by applying a stress with opposite sign (ferro-elasticity, FE).

In the last two decades, considerable attention has been devoted into developing macroscopic phenomenological SMA constitutive models to predict main features of SMA under thermo-mechanical loadings. These models are generally suitable for engineering applications due to their simplicity and compatibility with numerical methods, but they can only describe the global thermo-mechanical response while all microscopic details are ignored. One of the first one-dimensional (1-D) phenomenological models for SMAs was presented by Tanaka [3]. He considered exponential functions to describe phase transformations between detwinned martensite and austenite. Liang and Rogers [4] presented an alternative evolution law for the martensite volume fraction based on cosine functions. A major shortcoming for proposed models in [3,4] is that they are not able to simulate martensite orientation process which is responsible for the SME in low temperatures. Brinson [5] separated martensitic volume fraction into martensitic volume fractions induced by temperature and stress to include the effect of the martensite orientation. She developed a 1-D SMA constitutive equation and presented cosine phase transformation equations which can reproduce martensite transformation/orientation. Boyd and Lagoudas [6] developed a three-dimensional (3-D) constitutive model to simulate PE and SME. They used the volume fraction of martensite as internal variable and a transformation evolution equation to connect it with the transformation strain. Lagoudas et al. [7] provided

^{*} Corresponding author. Tel.: +98 21 64543429; fax: +98 21 66419736.
E-mail address: aghdam@aut.ac.ir (M.M. Aghdam).

a unified framework for the earlier models of Tanaka [3], Liang and Rogers [4], and Boyd and Lagoudas [6]. It should be mentioned that the proposed models in [3–7] are not able to simulate ferro-elasticity at low temperatures. Recently, Panico and Brinson [8] proposed a 3-D macroscopic phenomenological model which accounted for the effect of multiaxial stress states and non-proportional loading histories. Their model is able to reproduce main aspect of SMAs such as pseudo-elasticity, ferro-elasticity and martensite transformation/orientation/reorientation.

In recent years, some research works have been focused on the static and dynamic control of structures using SMA components. In the field of static analysis, Marfia et al. [9] investigated the static behavior of elastic beams with two integrated SMA layers under different thermo-mechanical loadings. They developed three finite element models based on the Euler–Bernoulli, Timoshenko, and layerwise theories to analyze SMA composite beams under thermo-mechanical static loads. Liew et al. [10] presented a numerical simulation for the pseudo-elastic behavior of a single SMA beam subjected to thermo-mechanical static loads using 3-D SMA constitutive model proposed by Boyd and Lagoudas [6]. They developed an explicit incremental displacement-based element-free Galerkin formulation by introducing moving least squares shape functions and continuum tangent stiffness tensors into a weak form of equilibrium equation in the referential configuration.

In the field of dynamic analysis, Gilat and Aboudi [11] investigated the dynamic response of plates, consisting of active polymeric/metallic matrices composites with embedded SMA fibers under a suddenly applied thermal loading using 3-D SMA model of Lagoudas et al. [7]. The effects of SMA thin films embedded in composite plates for improving damage resistance of composite structures under low velocity impact were numerically examined by Kim et al. [12]. 3-D SMA model proposed by Lagoudas et al. [7] was used to simulate the response of SMA thin films and was implemented using the user defined material subroutine of the ABAQUS/Explicit finite element program. Non-linear free vibration and damping of a clamped–clamped composite beam containing SMA wires with different pre-strains embedded in the mid-surface were investigated by Rezaei et al. [13]. 1-D Brinson model for SMA wires and Euler–Bernoulli beam theory for the displacement field were adapted and Galerkin method together with Runge–Kutta scheme was employed to solve the non-linear problem. Recently, dynamic response of a sandwich beam with SMA wire reinforced hybrid composite face sheets and flexible core was analyzed by Khalili et al. [14]. 1-D Brinson equation was employed to simulate pseudo-elastic behavior of SMA wires and a finite element method was developed to solve the non-linear governing equations.

The literature survey reveals that researchers generally employed either 1-D Brinson model [5] or 3-D model proposed by Lagoudas and co-workers [6,7] to predict SMA responses. However, as a stated above, these models are not able to reproduce ferro-elasticity effect of SMAs, which is responsible for accurate prediction of pre-strained SMA behavior under tension–compression stress at low temperatures. Therefore, the low temperature SMA response in the vibration control analysis of mechanical structures seems to be missing in the open literature.

The main objective of the present study is to address the geometrically non-linear vibration control of elastic beams subjected to the blast loads with integrated SMA layers in low and high temperatures. The equivalent single layer theory of Rayleigh–Euler–Bernoulli and von Karman geometrical non-linearity are used to describe displacement and strain fields of the SMA laminated composite beams. In order to predict SMA behavior, the 3-D Panico–Brinson model [8] is employed and simplified to the 1-D case. Analytical description of the 1-D SMA constitutive equation is presented. The model is able to simulate pseudo-elasticity, martensite transformation/orientation and in particular the ferro-elasticity

effects. Considering rotary inertia effects and using the Hamilton principle, an SMA composite beam element is first developed which is subsequently extended to the finite element equations of motion. Due to the phase transformation of SMA and consequently its material non-linearity, the equations of motion of SMA composite beam are coupled with the phase transformation kinetics of SMAs. Newmark and Newton–Raphson techniques along with a numerical algorithm are used to obtain an incremental solution of the coupled problem. In order to validate the proposed 1-D SMA model, the numerical results of partial uniaxial tension tests at different temperatures are compared with experimental data in the literature which show excellent correlations. After validation, a set of parametric study is performed to examine the influence of pre-strain, temperature, location and thickness of SMA layer on the transient response of SMA composite beams subjected to various blast pulses. Numerical results reveal that SMA layers with high pre-strain have a passive vibration control capability in low temperatures and lead to a better performance compared to the pseudo-elastic SMA layers.

2. Theoretical formulations

2.1. 1-D SMA constitutive model

In the present work, thin SMA layers are employed to control structure response. To simulate 1-D SMA behavior, the 3-D macroscopic phenomenological constitutive model recently proposed by Panico and Brinson [8] is recast to the 1-D uniaxial tension–compression case. The present model is developed within the theory of irreversible thermodynamics in the realm of a small-strain regime.

The total martensite fraction, ξ , can be expressed as the sum of temperature-induced and stress-induced parts:

$$\xi = \xi_T + \xi_S, \quad 0 \leq \xi \leq 1 \quad (1)$$

where ξ_T denotes the fraction of the material that is purely temperature-induced martensite with multiple variants, and ξ_S represents the fraction of the material that has been transformed by stress into a single martensitic variant.

A general assumption of additivity of total strain can be adopted in the form:

$$\varepsilon = \varepsilon_e + \varepsilon_{tr} \quad (2)$$

where ε_e represents the elastic strain and ε_{tr} denotes an inelastic strain due to the phase transformation from austenite or twinned martensite which is not associated to any macroscopic strain to the detwinned martensite associated to a macroscopic strain. Moreover, considering ε_L as the maximum transformation strain reached at the end of the transformation during a uniaxial tension test, one requires:

$$0 \leq |\varepsilon_{tr}| \leq \varepsilon_L \quad (3)$$

where ε_L can be regarded as a material parameter. Since ε_L is the amount of strain associated with the detwinned martensitic variant, then ε_{tr} can be related to the volumetric fraction ξ_S of such a variant by:

$$|\varepsilon_{tr}| = \xi_S \varepsilon_L \quad (4)$$

The increment of detwinned martensite fraction can be written from Eq. (4) as:

$$\dot{\xi}_S = \dot{\varepsilon}_{tr} \text{sgn}(\varepsilon_{tr}) / \varepsilon_L \quad (5)$$

where $\text{sgn}(\bullet)$ denotes signum function.

Considering the elastic strain ε_e and the absolute temperature, T , as control variables together with ξ_T and ξ_S as internal variables,

the following classical expression for the Helmholtz free energy function of the three-phase system is obtained as:

$$\begin{aligned} \psi(\varepsilon_e, T, \xi_s, \xi_T) = & \frac{1}{2\rho} E_s \varepsilon_e^2 + u_0^A - T s_0^A + \xi_T (T \Delta s_0 - \Delta u_0) \\ & + \xi_s (T \Delta s_0 - \Delta u_0) + c_v [T - T_0 - T \ln(T/T_0)] \\ & + \frac{1}{2} H_s \xi_s^2 \end{aligned} \quad (6)$$

where ρ is the material density, E_s is the elasticity modulus assumed to be the same for all phases. The energy and entropy differences $\Delta u_0 = u_0^A - u_0^M$ and $\Delta s_0 = s_0^A - s_0^M$ have been adopted where u_0^A and u_0^M are the specific free energies of austenite and martensite, respectively, and s_0^A and s_0^M are the specific entropies of austenite and martensite. u_0^M and s_0^M are considered to be the same for both detwinned and twinned martensite. Also, c_v is the specific heat at constant volume, and T_0 is the equilibrium temperature between parent and product phases. Moreover, H_s is a material parameter which governs the initial hardening during phase transformation. It should be mentioned that, in the energy contribution deriving from ξ_s the MacCauley bracket is introduced which calculate the positive part of its argument and defined as $\langle x \rangle = (x + |x|)/2$.

Following standard arguments [15], the constitutive equations can be extracted from Helmholtz free energy function (6) according to the second law of thermodynamics as:

$$\sigma = E_s (\varepsilon - \varepsilon_{tr}) \quad (7a)$$

$$X_s = \sigma \operatorname{sgn}(\varepsilon_{tr}) - \frac{\rho \Delta s_0}{\varepsilon_L} \langle T - \Delta u_0 / \Delta s_0 \rangle - \frac{\rho H_s}{\varepsilon_L} \xi_s \quad (7b)$$

$$X_T = -\frac{\rho \Delta s_0}{\varepsilon_L} (T - \Delta u_0 / \Delta s_0) \quad (7c)$$

where σ is the stress, while X_s and X_T are driving forces associated respectively to the detwinned and twinned martensite fractions.

For the internal variables ξ_s and ξ_T , limit functions are introduced as:

$$\begin{aligned} F_s &= X_s \operatorname{sgn}(X_s) - Y_s^{f/r}(\xi_s) \\ F_T &= X_T \operatorname{sgn}(X_T) - Y_T^{f/r}(\xi_T, \sigma) \end{aligned} \quad (8)$$

where $Y_s^{f/r}(\xi_s)$ and $Y_T^{f/r}(\xi_T, \sigma)$ are functions that direct kinetics of mechanically and thermally induced forward/reverse phase transformations, respectively. Finally, the consistency parameters and the limit functions satisfy the classical Kuhn–Tucker conditions as:

$$\begin{aligned} F_s &\leq 0 \quad \dot{\xi}_s = \text{or} \neq 0, \quad F_s \dot{\xi}_s = 0 \\ F_T &\leq 0 \quad \dot{\xi}_T = \text{or} \neq 0, \quad F_T \dot{\xi}_T = 0 \end{aligned} \quad (9)$$

Motivated by the experimental/numerical tests [8,16], the kinetics of stress-induced forward/reverse phase transformations can be written in an exponential form as:

$$Y_s^{f/r}(\xi_s) = \begin{cases} Y_{s0}^f (1 + \frac{1}{n} \ln(1 - \xi_s)) + Y_{s1}^f (-\frac{1}{n} \ln(1 - \xi_s)) & \text{for } \dot{\xi}_s > 0 \\ Y_{s0}^r (-\frac{1}{n} \ln(\xi_s)) + Y_{s1}^r (1 + \frac{1}{n} \ln(\xi_s)) & \text{for } \dot{\xi}_s < 0 \end{cases} \quad (10)$$

where $Y_{s0}^{f/r}$ and $Y_{s1}^{f/r}$ are respectively the values of $Y_s^{f/r}$ at $\xi_s = 0$ and $\xi_s = 1$, related to the beginning and end of transformation. Since the exponential formulae become singular at $\xi_s = 0$ and 1, the quantities $\xi_s = 1 - e^{-n}$ and $\xi_s = e^{-n}$ are given for the complete martensitic and austenitic states, respectively. The unknown positive parameter n can be determined according to a digit approximated for zero. In the present work, $n = 7$ is used which leads to the approximation 0.001 for zero.

The temperature-induced transformation kinetics, $Y_T^{f/r}$, can be also described through the following function [8]:

$$Y_T^{f/r}(\xi_T, \sigma) = \begin{cases} \frac{\rho c^f}{\varepsilon_L} \xi_T & \text{for } \dot{\xi}_T > 0 \\ \frac{Y_{T0}^r}{\varepsilon_L} + |\sigma| + \frac{\rho c^r}{\varepsilon_L} (1 - \xi_T) & \text{for } \dot{\xi}_T < 0 \end{cases} \quad (11)$$

where $c^{f/r}$ and Y_{T0}^r are coefficients that describe the kinetics of thermally induced forward/reverse phase transformations.

Finally, in order to update the transformation strain, the time-discrete system of equations can be written as:

$$I: \begin{cases} \sigma - E_s (\varepsilon - \varepsilon_{tr}) = 0 \\ (\varepsilon_{tr} - \varepsilon_{tr}^0) - (\xi_s - \xi_s^0) \operatorname{sgn}(\varepsilon_{tr}) \varepsilon_L = 0 \\ X_s = \sigma \operatorname{sgn}(\varepsilon_{tr}) - \frac{\rho \Delta s_0}{\varepsilon_L} \langle T - \Delta u_0 / \Delta s_0 \rangle - \frac{\rho H_s}{\varepsilon_L} \xi_s \\ F_s = X_s \operatorname{sgn}(X_s) - Y_s^{f/r}(\xi_s) \leq 0 \\ \text{Kuhn–Tucker condition: } F_s \leq 0, (\xi_s - \xi_s^0) = \text{or} \neq 0, F_s (\xi_s - \xi_s^0) = 0 \end{cases} \quad (12a)$$

$$II: \begin{cases} X_T = -\frac{\rho \Delta s_0}{\varepsilon_L} (T - \Delta u_0 / \Delta s_0) \\ F_T = X_T \operatorname{sgn}(X_T) - Y_T^{f/r}(\xi_T, \sigma) \leq 0 \\ \text{Kuhn–Tucker condition: } F_T \leq 0, (\xi_T - \xi_T^0) = \text{or} \neq 0, F_T (\xi_T - \xi_T^0) = 0 \\ \text{if } \xi_s + \xi_T > 1 \Rightarrow \dot{\xi}_T = -\dot{\xi}_s \end{cases} \quad (12b)$$

in which backward-Euler integration scheme was used to discretize the evolution equation. The system of equations *I* is uncoupled from *II* and system of equation *II* can be computed after solving *I* and specifying σ and ξ_s . In order to solve non-linear system of equation, an elastic-predictor inelastic-corrector return map algorithm can be implemented [17]. It should be mentioned that in the case of $\varepsilon_{tr} = 0$, it is not possible to create martensite phase. To remove this failing, $\operatorname{sgn}(\varepsilon_{tr})$ can be replaced by $\operatorname{sgn}(\sigma)$.

The parameters c^f, c^r, Y_{T0}^r and Δu_0 can be obtained in terms of Δs_0 and four characteristic temperatures by applying the Kuhn–Tucker condition on thermally induced forward/reverse phase transformations at zero stress as:

$$\begin{aligned} c^f &= \Delta s_0 (M_s^0 - M_f^0) \\ c^r &= \Delta s_0 (A_f^0 - A_s^0) \\ Y_{T0}^r &= \rho \Delta s_0 (A_s^0 - M_s^0) \\ \Delta u_0 &= \Delta s_0 M_s^0 \end{aligned} \quad (13)$$

In a similar way, by considering an applied stress, the Kuhn–Tucker condition on thermally induced reverse phase transformation yields to the slope of transformation line to austenite in the uniaxial phase kinetics diagram as:

$$C_A = \rho \Delta s_0 / \varepsilon_L \quad (14)$$

By knowing the parameter Δs_0 through entropic consideration and the mass density ρ , Eq. (13) allows determination of c^f, c^r, Y_{T0}^r and Δu_0 as a function of the characteristic temperatures. The parameter ε_L can be easily found from a uniaxial pseudo-elastic test while H_s and the kinetics parameters $Y_{s0}^f, Y_{s1}^f, Y_{s0}^r$ and Y_{s1}^r are calibrated in order to replicate the SMA hardening behavior during forward and reverse phase transformations.

2.2. Kinematics and constitutive relations

In the present research, the transient response of isotropic elastic beam subjected to the transverse impulse loads is controlled using SMA materials. To this end, SMA material in the form of layer is bonded on the surface or embedded in the elastic matrix which leads to the SMA composite beams. Consider a general three layered beam with the length L and width b consisting of two isotropic layers and an SMA layer embedded into the structure, as depicted in Fig. 1. By ignoring the top or bottom elastic layer, the case of SMA bonded on the surface of elastic beam is achieved.

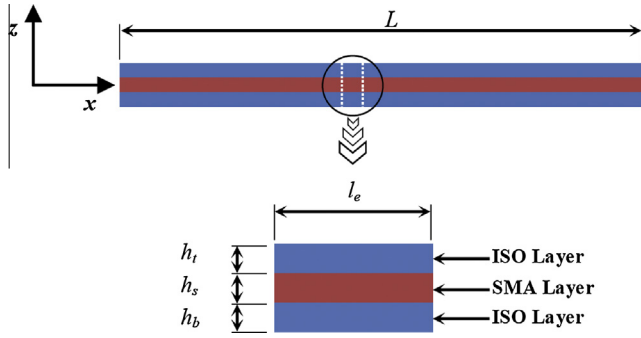


Fig. 1. Geometry and dimensions of an SMA laminated composite beam (ISO-SMA-ISO).

The thickness of the SMA, top and bottom layers are given as h_s , h_t and h_b , respectively. Cartesian coordinate system (x, z) is located on the mid-plane of structure where x is the beam axial direction, whereas z denotes the upward transverse direction. In Fig. 1, the abbreviation ISO represents isotropic layer and will be used henceforth.

Since the main purpose of the present analysis is to control the global dynamic response of thin SMA composite beams under blast loads, the equivalent single layer theory of Rayleigh–Euler–Bernoulli is used. The Rayleigh–Euler–Bernoulli laminated beam theory provide a sufficiently accurate description of global response for thin laminated structures [18,19]. Also, this beam model takes into account the effects of rotary inertia which is responsible for an accurate prediction of high frequency responses [19,20].

Based on the equivalent single-layer theory of Rayleigh–Euler–Bernoulli, the displacement components of a material point within the thin SMA composite beam domain in Cartesian coordinates system may be expressed as follows:

$$\begin{aligned}\bar{u}(x, z, t) &= u(x, t) - z w'(x, t) \\ \bar{w}(x, z, t) &= w(x, t)\end{aligned}\quad (15)$$

where $\bar{u}(x, z, t)$ and $\bar{w}(x, z, t)$ are displacements of an arbitrary point of the beam along the x and z directions, respectively, and u and w are the displacements of the mid-surface of the beam which are functions of x only. Also, the prime on a variable indicates its derivative with respect to x .

Assuming small strain and moderate rotation, the non-linear strain–displacement relation in von Karman sense can be expressed as:

$$\varepsilon = \bar{u}' + \frac{1}{2} w'^2 \quad (16)$$

Hooke law for the elastic layer can be written as:

$$\sigma = E_e \varepsilon \quad (17)$$

where E_e is the elasticity modulus of isotropic elastic medium.

2.3. Variational formulation

In this section, variational formulation for an SMA composite beam element is developed. In order to derive the governing equations of motion and boundary conditions for the smart composite beam, the Hamilton principle is utilized as:

$$\int_{t_0}^{t_1} (\delta K - \delta U + \delta W_{ex}) dt = 0 \quad (18)$$

where δ denotes the variation operator and $[t_0, t_1]$ describes arbitrary time interval. Also, δK denotes the virtual kinetic energy and δU is the virtual strain energy stored in the element due to actual

stress σ in moving through the virtual strain $\delta \varepsilon$. Furthermore, δW_{ex} is the work done by externally applied loads in moving through their respective virtual displacements.

The kinetic energy by considering rotary inertia effects and strain energy of the SMA composite beam can be written as:

$$\delta K = - \int_{V_e} \rho_e (\delta \bar{u}^T \ddot{u} + \delta w^T \ddot{w}) dV_e - \int_{V_s} \rho_s (\delta \bar{u}^T \ddot{u} + \delta w^T \ddot{w}) dV_s \quad (19a)$$

$$\delta U = \int_{V_e} \delta \varepsilon^T \sigma_e dV_e + \int_{V_s} \delta \varepsilon^T \sigma_s dV_s \quad (19b)$$

where the superscript T denotes transpose, while the superposed dot represents differentiation with respect to time. Also, V_e and V_s indicate the element volume related to SMA and elastic media, respectively. By considering the definition of stress in SMA and elastic media from Eqs. (7a) and (17), the strain energy can be rewritten as:

$$\delta U = \int_{V_e} E_e \delta \varepsilon^T \varepsilon dV_e + \int_{V_s} E_s \delta \varepsilon^T \varepsilon dV_s - \int_{V_s} E_s \delta \varepsilon^T (\varepsilon_{tr} - \varepsilon_0) dV_s \quad (20)$$

where ε_0 is the initial martensitic residual strain along the x direction (pre-strain) in the SMA medium.

The external work done by the distributed transverse load $q(x)$ and the generalized nodal forces including external axial (P_j, P_{j+1}) and transverse shear (V_j, V_{j+1}) forces and bending moment (M_j, M_{j+1}) on the first (j) and end ($j+1$) nodes of the element can be defined as:

$$\begin{aligned}\delta W_{ex} &= \int_l \delta w^T q(x) dx + \delta u_j^T P_j + \delta u_{j+1}^T P_{j+1} + \delta w_j^T V_j \\ &\quad + \delta w_{j+1}^T V_{j+1} + \delta \theta_j^T M_j + \delta \theta_{j+1}^T M_{j+1}\end{aligned}\quad (21)$$

where u_j , w_j and u_{j+1} , w_{j+1} are axial and transverse displacements of the first and the end nodes of the element while parameter θ refers to $-w'$.

2.4. Finite element model

The weak form of the equations of motion of SMA composite beam element presented in previous section is used here to develop the finite element formulation. To this end, the axial displacement $u(x, t)$ and transverse deflection $w(x, t)$ are interpolated as:

$$u(x, t) = \sum_{i=1}^2 u_i(t) \varphi_i(x); \quad w(x, t) = \sum_{i=1}^4 w_i(t) \psi_i(x) \quad (22)$$

where φ_i are the linear Lagrange interpolation functions, and ψ_i are the Hermite cubic interpolation functions [21]. Also, u_i and w_i represent mechanical nodal values.

Using the proposed function for displacement field components, these quantities can be expressed in terms of mechanical nodal variables as:

$$\begin{aligned}\bar{u} &= \mathbf{N}(x, z) \mathbf{u}_E(t) \\ \bar{w} &= \mathbf{M}(x) \mathbf{u}_E(t)\end{aligned}\quad (23)$$

where $\mathbf{N}$ and $\mathbf{M}$ are the element shape function vectors, and $\mathbf{u}_E$ is the vector of element nodal variables defined as:

$$\mathbf{u}_E = \{u_j \quad w_j \quad \theta_j \quad u_{j+1} \quad w_{j+1} \quad \theta_{j+1}\}^T \quad (24)$$

According to the strain–displacement relation (16), the strain field of the SMA composite beam element can be rewritten as:

$$\varepsilon = \mathbf{N}' u_E + \frac{1}{2} (\mathbf{M}' u_E)^2 = \bar{\mathbf{N}}(x, z, \mathbf{u}_E) \mathbf{u}_E \quad (25)$$

The variation of the strain can be also expressed as:

$$\delta \varepsilon = \bar{\mathbf{N}}(x, z, \mathbf{u}_E) \delta \mathbf{u}_E \quad (26)$$

Using the displacement and strain fields in the discretized form (23), (25), and (26), the kinetic and strain energies of the SMA composite beam element can be rewritten as:

$$\begin{aligned} \delta K &= -\delta \mathbf{u}_E^T \int_{V_e} \rho_e (\mathbf{N}^T \mathbf{N} + \mathbf{M}^T \mathbf{M}) \ddot{\mathbf{u}}_E dV_e - \delta \mathbf{u}_E^T \int_{V_s} \rho_s (\mathbf{N}^T \mathbf{N} + \mathbf{M}^T \mathbf{M}) \ddot{\mathbf{u}}_E dV_s \\ \delta U &= \delta \mathbf{u}_E^T \int_{V_e} E_e \tilde{\mathbf{N}}^T \tilde{\mathbf{N}} \mathbf{u}_E dV_e + \delta \mathbf{u}_E^T \int_{V_s} E_s \tilde{\mathbf{N}}^T \tilde{\mathbf{N}} \mathbf{u}_E dV_s - \delta \mathbf{u}_E^T \int_{V_s} E_s \tilde{\mathbf{N}}^T (\varepsilon_{tr} - \varepsilon_0) dV_s \end{aligned} \quad (27)$$

It should be noted that the transformation strain ε_{tr} is variable through the x and z directions. However, it is assumed that ε_{tr} to be constant along the SMA element length while the Gauss–Legendre numerical integration rule is utilized to evaluate the through-the-thickness integral consists of ε_{tr} .

By substituting the kinetic and strain energies (27) and the external virtual work (21) into the Hamilton principle (18), the governing equations of motion are obtained as:

$$\mathbf{M}_E \ddot{\mathbf{u}}_E + \mathbf{K}_E(\mathbf{u}_E, \mathbf{u})_E = \mathbf{f}_E^s(\varepsilon_{tr}, \mathbf{u}_E) + \mathbf{f}_E \quad (28)$$

where

$$\mathbf{M}_E = \int_{V_e} \rho_e (\mathbf{N}^T \mathbf{N} + \mathbf{M}^T \mathbf{M}) dV_e + \int_{V_s} \rho_s (\mathbf{N}^T \mathbf{N} + \mathbf{M}^T \mathbf{M}) dV_s \quad (29a)$$

$$\mathbf{K}_E = \int_{V_e} E_e \tilde{\mathbf{N}}^T \tilde{\mathbf{N}} dV_e + \int_{V_s} E_s \tilde{\mathbf{N}}^T \tilde{\mathbf{N}} dV_s \quad (29b)$$

$$\mathbf{f}_E^s = \int_{V_s} E_s \tilde{\mathbf{N}}^T (\varepsilon_{tr} - \varepsilon_0) dV_s \quad (29c)$$

$$\mathbf{f}_E = \int_l \mathbf{M}^T q(x) dx + \{P_j V_j M_j P_{j+1} V_{j+1} M_{j+1}\}^T \quad (29d)$$

Eq. (28) is the non-linear governing equation of SMA composite beam element. Finally, Eq. (28) can be utilized to generate global finite element vectors and matrices by assembly and imposition of boundary conditions which results in:

$$\mathbf{M} \ddot{\mathbf{u}} + \mathbf{K}(\mathbf{u}) \mathbf{u} = \mathbf{f}^s(\varepsilon_{tr}, \mathbf{u}) + \mathbf{f} \quad (30)$$

Eqs. (12) and (30) show that the governing equations of motion of SMA composite beam and the phase transformation kinetics are coupled together, which makes the problem to be more complicated. This system of equations will be solved for the dynamic analysis of SMA composite beams in the next section.

2.5. Time-discrete frame and solution algorithm

In this section, the coupled governing equations of the present problem described in previous sections in particular Eqs. (12) and (30) are solved to analyze the dynamic behavior of SMA composite beams under impulsive loads. To this end, the Newmark direct integration method [21] is employed to discretize the governing equations of motion (30) in the time domain which leads to a system of non-linear algebraic equations. The resulted non-linear equations are then solved using the Newton–Raphson iterative method [21] while the quantity of inelastic transformation strain in Eq. (30) is updated through SMA constitutive equations (12) in every step of the solution. The numerical solution procedure can be summarized as:

- (1) Data predictor: $\mathbf{u}^{(n+1)} = \mathbf{u}^{(n)}$; $\mathbf{f}^{s(n+1)} = \mathbf{f}^{s(n)}$; $\mathbf{f}^{(n+1)}$.
- (2) Compute $\mathbf{u}^{(n+1)}$; $\dot{\mathbf{u}}^{(n+1)}$; $\ddot{\mathbf{u}}^{(n+1)}$ by solving the equations of motion.

If the prescribed tolerance is satisfied then $n \rightarrow n+1$ and go to step (1). Else:

- (a) Check transformation/orientation:
 - (I) No. Elastic step: update $\mathbf{u}^{(n+1)}$; $\mathbf{f}^{s(n+1)}$ and then return to step (2).
 - (II) Yes. According to the fixed strain transformation, compute SMA internal variables and inelastic transformation strain and update $\mathbf{u}^{(n+1)}$; $\mathbf{f}^{s(n+1)}$ and then return to step (2).

2.6. Transverse blast loads

Some commonly time-dependent impulsive loads, as presented in Fig. 2, are considered. The definition of transverse blast loads $V(t)$ in the time domain is summarized as:

$$(a) \text{ Rectangular pulse : } V(t) = \begin{cases} V & 0 \leq t \leq \tau \\ 0 & \tau < t \end{cases} \quad (31a)$$

$$(b) \text{ Half-sine pulse : } V(t) = \begin{cases} V \sin(\pi t / \tau) & 0 \leq t \leq \tau \\ 0 & \tau < t \end{cases} \quad (31b)$$

$$(c) \text{ Incremental pulse : } V(t) = \begin{cases} Vt/\tau & 0 \leq t \leq \tau \\ 0 & \tau < t \end{cases} \quad (31c)$$

$$(d) \text{ N-shaped pulse : } V(t) = \begin{cases} V(1 - t/\tau) & 0 \leq t \leq r\tau \\ 0 & r\tau < t \end{cases} \quad (31d)$$

in which V is the magnitude of the imposed transverse impulse load and τ denotes the loading duration. It is worth mentioning that sonic boom effect could be modeled as an N-shaped pulse described as Eq. (31d). Parameter r in Eq. (31d) denotes the shock pulse length factor. For $r = 1$, the N-shaped pulse degenerates into a triangular pulse, for $r = 2$, a symmetric sonic boom pulse is obtained whereas $r \neq 2$ corresponds to an asymmetric N-shaped pulse.

3. Numerical results and discussion

In this section, capabilities of the present 1-D SMA model are first examined to simulate experimental results available in the open literature. After validating the proposed SMA model, to assess the performance of SMA layers in controlling the non-linear transient response of elastic beams under blast loads, extensive numerical results are presented. Unless otherwise stated, in all the simulations discussed in this section, the SMA material is assumed to be initially in parent phase ($\xi_S = \xi_T = 0$) for temperatures $T \geq M_s^0$.

3.1. SMA model predictions

In order to examine capability of the developed 1-D model in reproducing main features of SMAs, the experimental data available at Roundrobin SMA modeling website [16] are employed and compared with the present numerical results in Fig. 3. This figure explores the stress–strain response of an NiTi wire under uniaxial partial tension test at different temperatures. The material parameters defined in the present model are calibrated to match the uniaxial stress–strain response as illustrated in Fig. 3 and together with the transformation temperatures are listed in Table 1 for case A. As can be seen in Fig. 3a, the specimen in the temperature 253 K is first loaded to achieve 4% strain and then unloaded which leads to a residual strain at the end of the first cycle. Once again, the load increases up to 8% strain. Finally, unloading to zero stress yields to a residual strain. Similar processes are adapted as shown in Fig. 3b and c, respectively, for temperatures 283 K and 293 K but unloading stage in the first cycle is restricted to 2% strain. Fig. 3 reveals that the main features of SMA wire is well simulated in capturing martensite transformation, internal loops induced by partial loading–unloading and pseudo-elastic response.

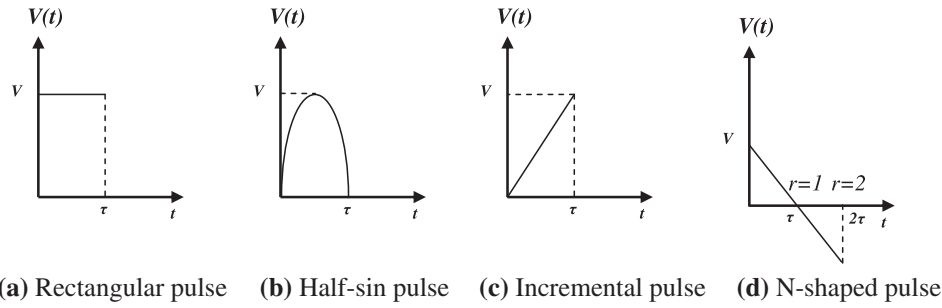


Fig. 2. Load pulse shapes.

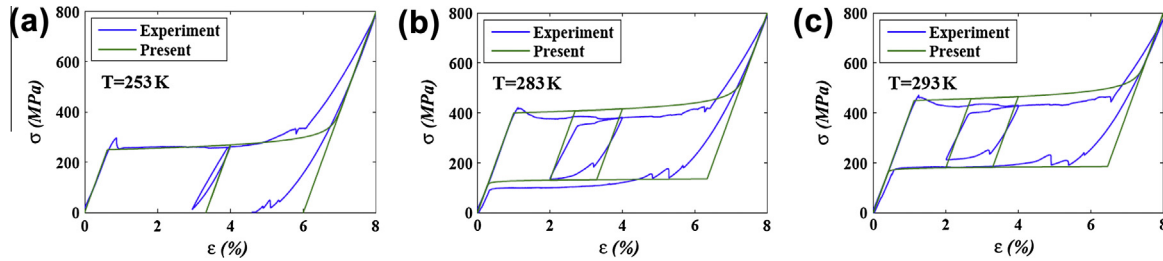


Fig. 3. Model predictions compared to the experiment [16] for partial tension test at different temperatures.

Table 1
SMA material parameters used in the numerical tests.

Parameter	Case A	Case B ^a	Unit
E	40,000	68,400	MPa
ν	0.33	0.36	–
ρ	6500	8000	kg/m ³
ϵ_L	0.060	0.038	–
Δu_0	11,585	14,725	J/kg
Δs_0	46.15	47.50	J/kgK
H_s	9.23	100	J/kg
Y_{s0}^f	240	88.92	MPa
Y_{s1}^f	379	173.44	MPa
Y_{s0}^r	45	90	MPa
Y_{s1}^r	26	91.05	MPa
Y_{T0}^r	1.5	2.66	MPa
c^f	92.31	190	J/kg
c^r	184.61	95	J/kg
M_f^0	249	306	K
M_s^0	251	310	K
A_s^0	256	317	K
A_f^0	260	319	K

^a Adapted from Ref. [8].

characteristic temperatures are given in Table 1 for the case B, adapted from Ref. [8]. Fig. 4a shows that the residual strain obtained after the first unloading to zero stress is rather significant at temperature 310 K which is equal to martensite start temperature. At a higher temperature 318 K, which is higher than austenite start temperature and lower than austenite finish temperature, the residual strain decreases as shown in Fig. 4b. This residual strain vanishes at 325 K which is above the austenite finish temperature and the SMA exhibits pseudo-elastic response, see Fig. 4c. It can be observed in Fig. 4a and b that the residual strain obtained from the first tensile loading–unloading stage is recovered by applying a stress with opposite sign (i.e., compressive stress). This phenomenon refers to the ferro-elasticity effect of SMAs. Finally, Fig. 4 indicates the model capability to reproduce martensite transformation and ferro-elasticity as found in Fig. 4a and b and pseudo-elastic response as shown in Fig. 4c.

In the next section, it would be found that the modeling of ferro-elasticity effect of SMAs is an important key feature to introduce SMA materials for the purpose of passive vibration control strategy.

3.2. Dynamic analysis of SMA composite beam

In order to analyze geometrically non-linear transient response of SMA composite beams under impulsive loads, a fully clamped composite beam with length 200 mm, width 20 mm and total

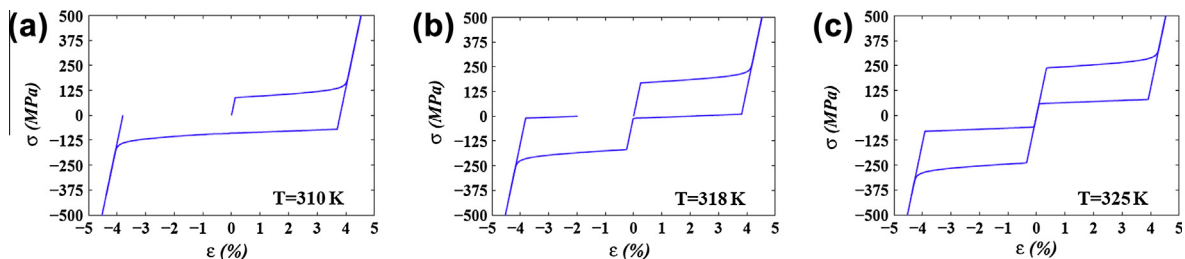


Fig. 4. Stress–strain response for an SMA specimen under uniaxial tension–compression test at different temperatures.

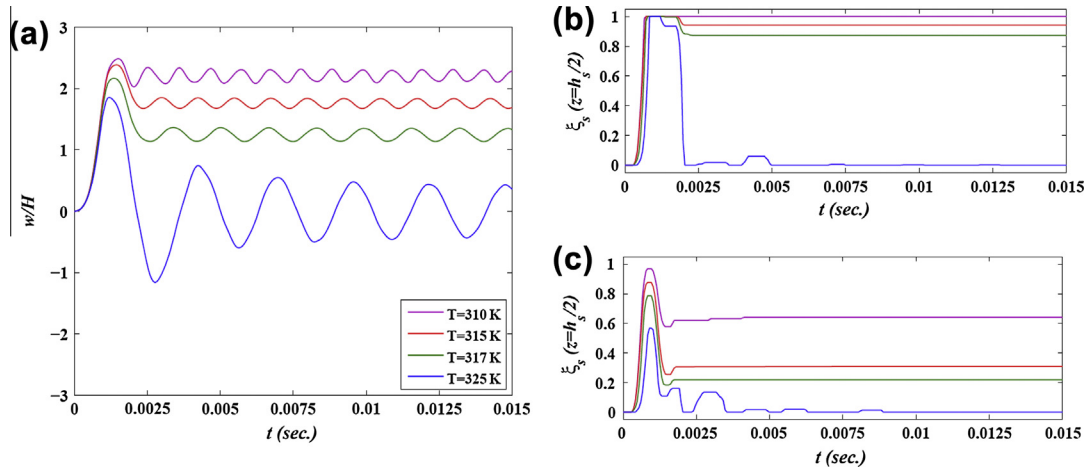


Fig. 5. The time-histories of the central deflection (a), detwinned martensite fraction at top point of beam mid-span (b) and detwinned martensite fraction at bottom point of beam mid-span (c) for an SMA beam at different temperatures.

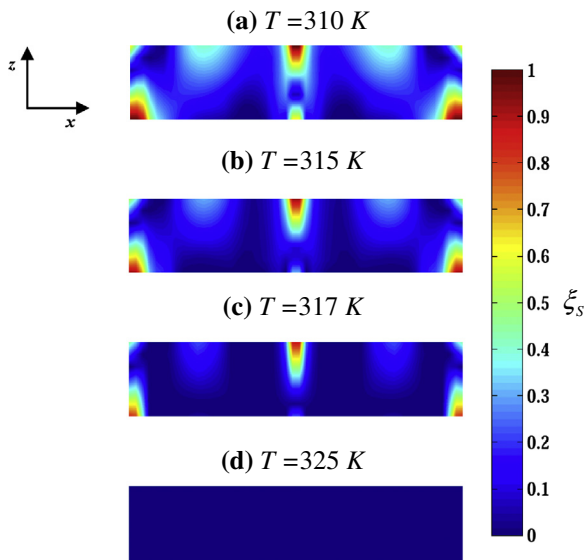


Fig. 6. Distribution of detwinned martensite fraction in SMA domain related to Fig. 5 at $t = 15$ ms in the undeformed body.

thickness $H = 5$ mm is considered. It is assumed that the elastic layer is made of Aluminum ($E_e = 70$ GPa, $\rho_e = 2707$ kg/m³). The material properties of the SMA are listed in Table 1 for the case B. In order to active the SMA layer, SMA is only heated and its temperature is uniformly raised [2]. On the other hand, it is assumed that the temperature of elastic layers remains in the reference temperature as a generally well-known assumption since the effect of elevated SMA temperature on adjacent elastic layers is negligible [2,22]. In all dynamical simulations, unless otherwise stated, it is assumed that the structure is subjected to an incremental impulse point load with the magnitude of 200 kN and loading duration $\tau = 1$ ms which is imposed upward on the center of the structure. Moreover, in all computations, 50 elements in the axial direction with 5 through-the-thickness Gauss points per element were used to obtain the converged results up to four significant digits.

Fig. 5 illustrates the central transverse displacement and detwinned martensite fraction time histories of a single SMA beam at different temperatures. Fig. 5b and c shows that detwinned martensite fractions at top and bottom surfaces have no identical value at loading step which is due to the geometrical non-linearity. As shown in Fig. 5a, the temperature has a significant effect on the response of the SMA beam so that high permanent displacement remains in the structure after unloading at temperatures below A_f^0 .

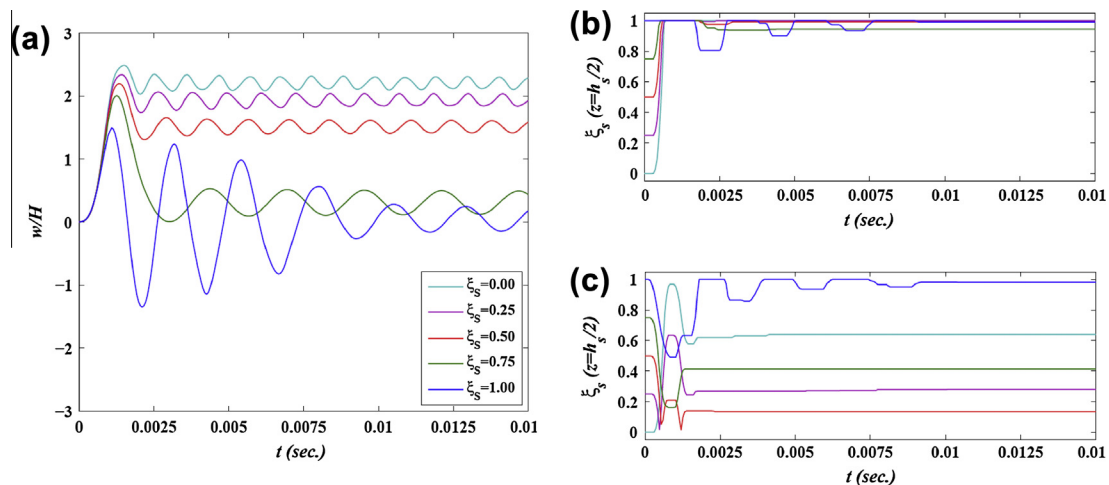


Fig. 7. The time-histories of the central deflection (a), detwinned martensite fraction at top point of beam mid-span (b) and detwinned martensite fraction at bottom point of beam mid-span (c) for an SMA beam with different initial values of detwinned martensite fraction.

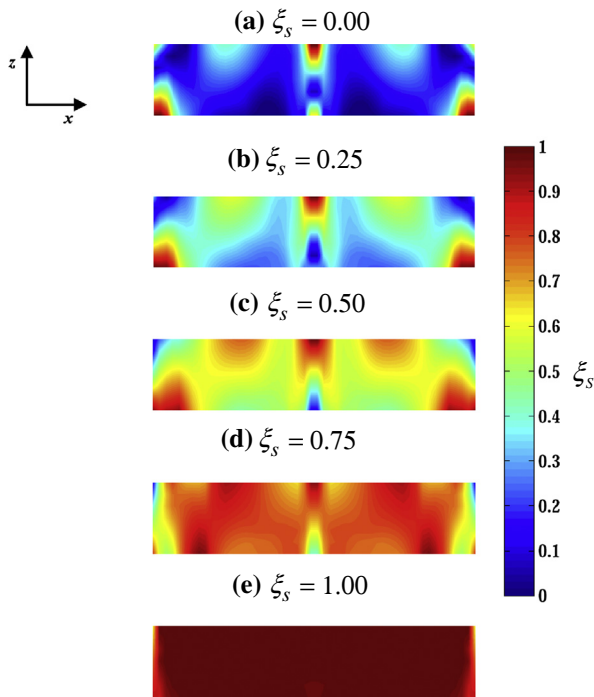


Fig. 8. Distribution of detwinned martensite fraction in SMA domain related to Fig. 7 at $t = 15$ ms in the undeformed body.

However, it is seen that the vibration amplitude of the SMA beam at temperature above A_f^0 reduces in the free motion regime ($t > \tau$). Fig. 5 reveals that the time histories of transverse displacement and detwinned martensite fraction of the SMA beam have an active influence on damping of the structure response while the SMA phase is transformed. Due to this fact, the damping effect of SMA materials disappears when no phase transformation occurs.

2-D contour plots of the detwinned martensite fraction of SMA beam related to Fig. 5 is depicted in Fig. 6 at $t = 15$ ms in the undeformed body. It can be concluded from Figs. 5 and 6 that the reverse phase transformation does not complete at low temperatures so that it leads to residual inelastic strain and reduction of SMA damping effect. Finally, it is found that maximum damping capacity of SMA materials with initial austenite phase can be achieved at high temperatures so that the final phase coincides on the initial phase.

The effect of pre-strain on the response of a single SMA beam is analyzed in Figs. 7 and 8. In order to achieve a pre-strain, the SMA

beam initially in austenite phase is first loaded in the axial direction and then unloaded and constrained by edge supports.

Fig. 7 illustrates the time histories of the transverse displacement and detwinned martensite fraction at the center of SMA beam with different pre-strains at temperature 310 K. As can be found from Fig. 7b, during the forced motion regime ($t < \tau$), the SMA beams except fully martensitic SMA beam undergo forward phase transformation at top surface due to the high tensile stress at this point. On the other hand, Fig. 7c reveals that the SMA beams except fully austenitic SMA beam experience initially reverse phase transformation at bottom surface due to the presence of compressive stress at this point which refers to SMA ferro-elasticity effect. Therefore, it can be found that the modeling of ferro-elasticity effect is essential to accurately predict the SMA response when the pre-strained SMA layers undergo a compressive stress. As can be found from Fig. 7c, detwinned martensite at the bottom surface of SMA beams with initial detwinned martensite fraction of 0.25 and 0.5 has a decreasing-increasing trend in the forced motion regime. In fact, detwinned martensite at the bottom surface is first reduced by developing the compressive stress and then it increases due to the presence of tensile stress at this point since all beam section is stretched in large deformation regime. As can be seen from Fig. 7a, in the free motion regime ($t > \tau$), the oscillation amplitude of SMA beams, unlike elastic beams, does not only depend on the distance from the equilibrium condition at end of forced motion range ($t = \tau$). Fig. 7b shows that the high value of pre-strain has a significant effect on the dynamic response and damping of SMA beams at low temperatures. This figure reveals that the use of SMA layers with initial detwinned martensite fraction more than or equal to 0.75 can result in damping of structure response in low temperatures. In fact, continuous transformation between martensite and austenite, as shown in Fig. 7b and c, produces hysteresis loops and dissipates mechanical energy while reduces vibration amplitude. This phenomenon is detected for the first time and not hitherto reported in the open literature. Since this phenomenon occurs in low temperatures and requires no electrical current to induce thermally driven transformation, it can be assigned to passive vibration control capability of SMAs rather than their active vibration control ability in high temperatures.

Finally, it is worth comparing Figs. 5a and 7a, which shows that the damping effect of SMA beam initially in detwinned martensite phase at low temperatures is more than that of SMA beam initially in austenite state at high temperatures.

Fig. 8 illustrates 2-D contour plots of the final detwinned martensite fraction of SMA beam related to Fig. 7 at $t = 15$ ms in the undeformed body. Figs. 7b, 7c and 8 show that the phase transformation does not continuously occur at low pre-strains which leads

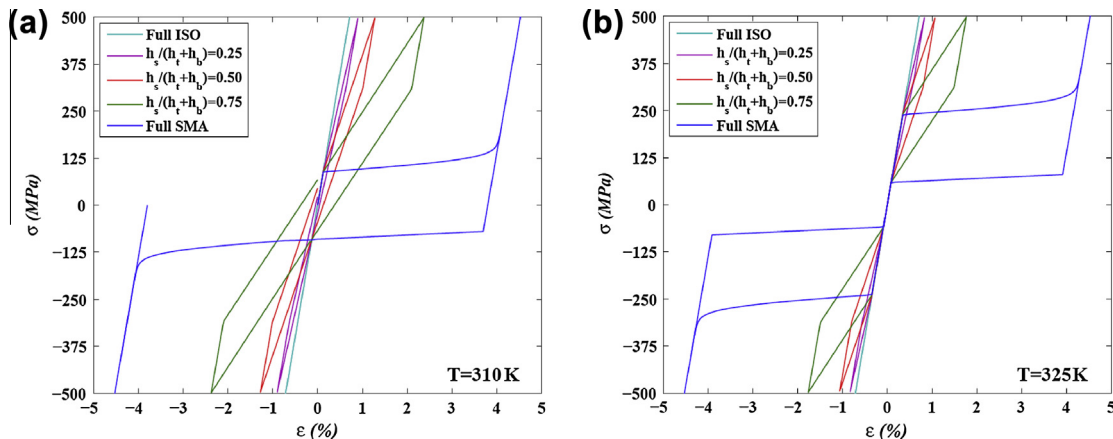


Fig. 9. Uniaxial stress-strain response of an SMA composite beam (ISO-SMA-ISO) under tension-compression test at different SMA temperatures.

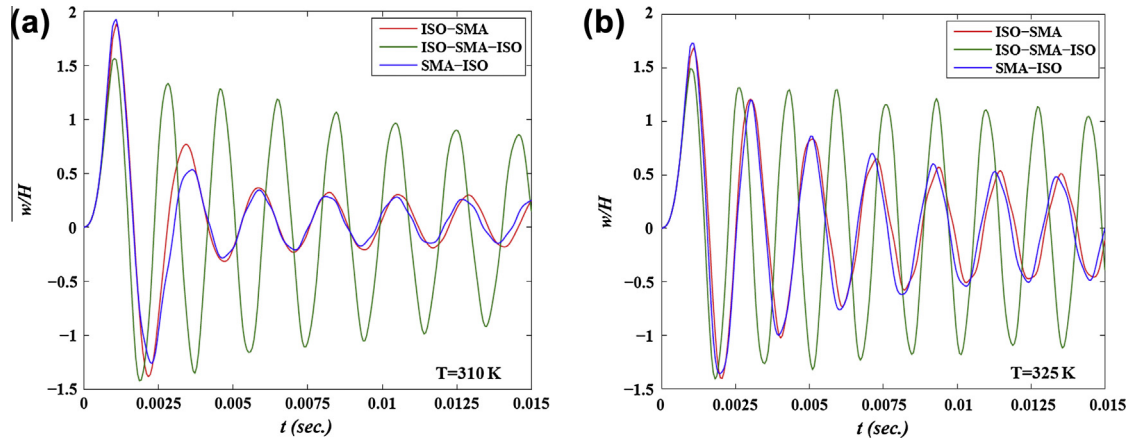


Fig. 10. Central deflection time-history of an SMA composite beam with various lamination schemes at different SMA temperatures.

to residual inelastic strain and affects SMA damping performance. The presented results in Figs. 7 and 8 reveal that maximum damping capacity of the SMA material at low temperatures is achieved for high value of pre-strain so that the final phase coincides on the initial phase.

In the following, the behavior of SMA composite beams consist of SMA and elastic layers are investigated under various static and dynamic loadings.

Fig. 9 depicts the simulation of uniaxial stress-strain response of a composite beam with the laminate stacking sequence of ISO-SMA-ISO and different SMA temperatures $T = M_s^0$ and $T > A_f^0$. The results are presented for different thickness ratios while the total thickness remains constant. Fig. 9 reveals that, although SMA and ISO layers have a close elasticity modulus, ISO

layers have a dominant role on the behavior of the composite beam. This is due to the fact that, replacing SMA material with ISO material descends considerably the plateau phenomenon and consequently the inelastic strain of the structure.

The effect of arrangement of SMA and ISO layers on the transient response of SMA composite beams with constant total thickness is investigated in Fig. 10 for two different SMA temperatures. It is assumed that all layers have an identical thickness in each specific arrangement. As explored from the presented results in Figs. 5 and 7, the single SMA layers initially in fully detwinned martensite and austenite states have a maximum damping capacity in low and high temperatures, respectively. Due to this fact, SMA layers related to the present example are considered to be initially in detwinned martensite and austenite phases at temperatures

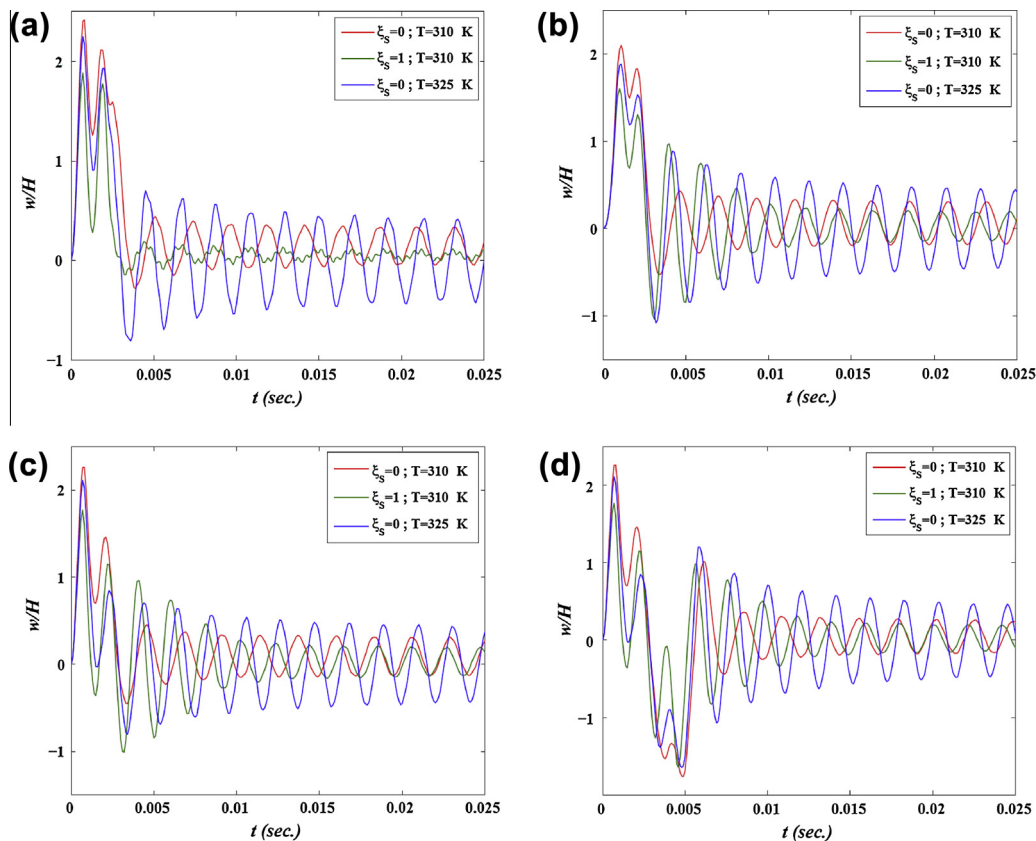


Fig. 11. Central deflection time-history of an SMA composite beam (SMA-ISO) subjected to: (a) rectangular pulse, (b) half-sin pulse, (c) triangular pulse ($r = 1$), (d) N-shaped pulse ($r = 2$).

310 K and 325 K, respectively. The primary inference drawn from Fig. 10 is SMA–ISO beam has a larger vibration amplitude in the forced motion range compared to ISO–SMA beam. When the impulsive load is imposed upward on the structure center, the SMA layer bonded on the top surface of elastic beam experiences tensile stress during the forced motion regime. On the other hand, in ISO–SMA structure, the SMA layer installed on the bottom surface of elastic beam undergoes first compressive and then tensile stress states due to the large deformation regime. A similar behavior was observed for fully SMA beams in Fig. 7. The difference in SMA stress state affects the phase transformation and consequently the structure response. As can be found from Fig. 10, inducing a larger stress in SMA layer of SMA–ISO beam increases considerably the plateau phenomenon and consequently the structure response softens more in comparison with ISO–SMA beam. Fig. 10 indicates that composite beams with surface-bonded SMA layers have a maximum deflection time-history in the forced motion range ($t < \tau$) while embedding the SMA layer into the structure leads to minimum deflection time-history in this interval. On the other hand, however, it is found that the mentioned arrangements, respectively, lead to minimum and maximum deflection time-histories in the free motion range ($t > \tau$). It can be concluded from comparing Fig. 10a and 10b that the composite beams with fully detwinned martensite SMA layers in the low temperature have a more damping capacity than those with fully austenite SMA layers in the high temperature. A similar conclusion was also achieved for single SMA beams as presented in Figs. 5a and 7a. These observations are detected for the first time and can be used as benchmark in the field of passive vibration control in future studies.

The time history of central deflection of composite beams with staking sequence of SMA–ISO subjected to different time-dependent blast pulses with loading duration 3 ms are demonstrated in Fig. 11. It is assumed that SMA layers at temperature 310 K can be initially in detwinned martensite or austenite phase. The presented results in Fig. 10 for incremental pulse and in Fig. 11 for other types of pulses indicate that SMA layers with conditions of $\xi_s = 1$, $T = 310$ K; $\xi_s = 0$, $T = 310$ K and $\xi_s = 0$, $T = 325$ K yield to minimum, intermediate and maximum deflection time histories in the free motion regime. It can be found that the passive vibration control strategy can play more effective role toward damping out the oscillations of the structure exposed to blast loads.

4. Conclusion

The present research was dedicated to study the vibration control capability of SMAs in the layer form at low temperatures. SMA layers were integrated to elastic beams subjected to blast loads for the vibration control strategy. The 3-D macroscopic phenomenological model proposed by Panico and Brinson [8] was used and recast to 1-D SMA model. Accordingly, a time-discrete counterpart of the non-linear 1-D SMA model was introduced, integrating the flow rule using an implicit backward-Euler scheme. Afterwards, the solution of the discrete model was performed by an elastic-predictor inelastic-corrector return map procedure. The equivalent single layer theory of Rayleigh–Euler–Bernoulli with capability of simulating rotary inertia effects together with von Karman geometrical non-linearity were used to develop the finite element equations of motion for SMA composite beams utilizing the Hamilton principle. Due to the phase transformation of SMAs, the equations of motion of SMA composite beams were highly coupled with the 1-D SMA constitutive model. The non-linear governing equations of motion were solved by using Newmark and Newton–Raphson methods and quantities including inelastic transformation strain were updated through SMA constitutive

equations in every step of the solution. In the numerical results and discussion section, the accuracy of the proposed 1-D SMA model was first verified through comparative study between the present model and experimental data on the partial uniaxial tension tests at different temperatures which revealed excellent correlations. Then, a set of parametric study was carried out to show the influence of pre-strain, temperature, location and thickness of SMA layers on the vibration control of elastic beams subjected to various blast pulses. The presented results reveal that the modeling of SMA ferro-elasticity feature is an essential tool for an accurate prediction of behavior of pre-stained SMA layers at low temperatures. It is concluded that SMA layers with high pre-strain at low temperatures have a passive vibration control capability and yield a better efficiency in comparison with pseudo-elastic SMA layers. This phenomenon is detected for the first time which may be found useful for studies in the field of active/passive vibration control.

References

- [1] Yamauchi K, Ohkata I, Tsuchiya K, Miyazaki S. Shape memory and superelastic alloys: technologies and applications. Cambridge: Woodhead Publishing Limited; 2011.
- [2] Lagoudas DC. Shape memory alloys: modeling and engineering applications. New York: Springer; 2008.
- [3] Tanaka K. A thermomechanical sketch of shape memory effect: one-dimensional tensile behavior. Res Mech 1986;18:251–63.
- [4] Liang C, Rogers CA. One-dimensional thermomechanical constitutive relations for shape memory materials. J Intell Mater Syst Struct 1990;1:207–34.
- [5] Brinson LC. One-dimensional constitutive behavior of shape memory alloys: thermomechanical derivation with non-constant material functions and redefined martensite internal variable. J Intell Mater Syst Struct 1993;4:229–42.
- [6] Boyd JG, Lagoudas DC. A thermodynamical constitutive model for shape memory materials. Part I: the monolithic shape memory alloy. Int J Plast 1996;12(6):805–42.
- [7] Lagoudas DC, Bo Z, Qidwai MA. A unified thermodynamic constitutive model for SMA and finite element analysis of active metal matrix composites. Mech Compos Mater Struct 1996;3:153–79.
- [8] Panico M, Brinson LC. A three-dimensional phenomenological model for martensite reorientation in shape memory alloys. J Mech Phys Solids 2007;55(11):2491–511.
- [9] Marfia S, Sacco E, Reddy JN. Superelastic and shape memory effects in laminated shape-memory-alloy beams. AIAA J 2003;41(1):100–9.
- [10] Liew KM, Ren J, Kitipornchai S. Analysis of the pseudoelastic behavior of a SMA beam by the element-free Galerkin method. Eng Anal Bound Elem 2004;28:497–507.
- [11] Gilat R, Aboudi J. Dynamic response of active composite plates: shape memory alloy fibers in polymeric/metallic matrices. Int J Solids Struct 2004;41: 5717–31.
- [12] Kim EH, Lee I, Roh JH, Bae JS, Choi IH, Koo KN. Effects of shape memory alloys on low velocity impact characteristics of composite plate. Compos Struct 2011;93:2903–9.
- [13] Rezaei H, Kadkhodaei M, Nahvi H. Analysis of nonlinear free vibration and damping of a clamped-clamped beam with embedded pre-strained shape memory alloy wires. J Intell Mater Syst Struct 2012;23(10):1107–17.
- [14] Khalili SMR, Botshekanan Dehkordi M, Carrera E, Shariyat M. Non-linear dynamic analysis of a sandwich beam with pseudoelastic SMA hybrid composite faces based on higher order finite element theory. Compos Struct 2013;96:243–55.
- [15] Truesdell C, Noll W. The non-linear field theories of mechanics. Berlin: Springer; 1965.
- [16] Sittner P, Pilch J, Heller L. 2009. <<http://ofm.fzu.cz/ofm/index.php/en/roundrobin>>.
- [17] Simo JC, Hughes TJR. Computational inelasticity. New York: Springer; 1998.
- [18] Reddy JN. An evaluation of equivalent-single-layer and layerwise theories of composite laminates. Compos Struct 1993;25:21–35.
- [19] Reddy JN. Mechanics of laminated composite plates and shells: theory and analysis. Boca Raton: CRC Press; 2004.
- [20] Han SM, Benaroya H, Wei T. Dynamics of transversely vibrating beams using four engineering theories. J Sound Vib 1999;225(5):935–88.
- [21] Reddy JN. An introduction to nonlinear finite element analysis. New York: Oxford University Press Inc.; 2004.
- [22] Hartl DJ, Lagoudas DC, Calkins FT. Advanced methods for the analysis design and optimization of SMA-based aerostructures. Smart Mater Struct 2011;20:094006 (20pp).